

MIAPbP Workshop Precision Lepton Physics

# $(g - 2)_\mu$ theory overview

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(he/him)

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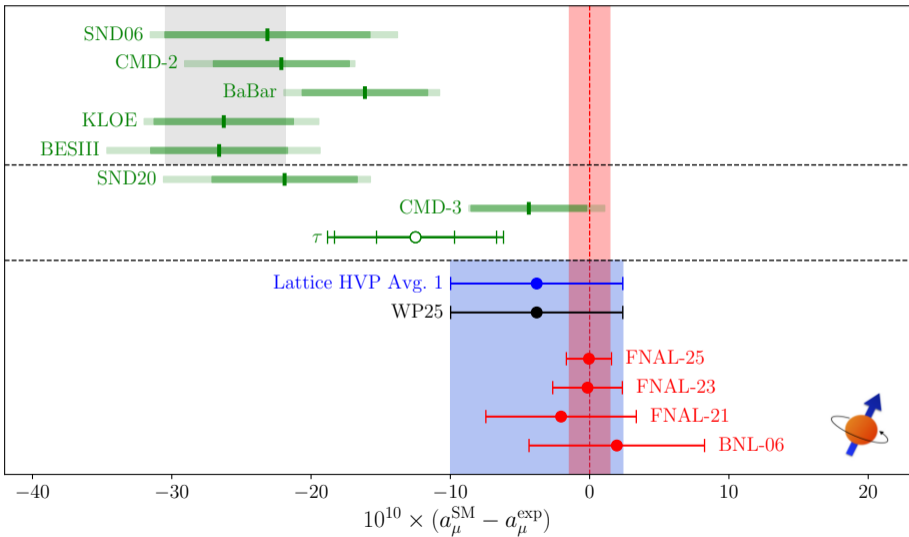
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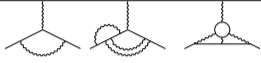
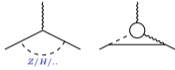


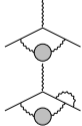
**slides:** [talks.yannickulrich.com/26-04-27-miapbp.pdf](https://talks.yannickulrich.com/26-04-27-miapbp.pdf)

- Antonio has said much already
- I'll focus on some things he skipped
- my area of interest: Monte Carlo for HVP data extractions
- **not** a lattice expert
- for more information than I could ever cover  $\Rightarrow$  see [\[WP25\]](#)

I'm not speaking for TI!

Figure 83



	value $\times 10^{10}$	example diagrams	
QED	11 658 471.88(2)		(7.27)
EW	15.44(4)		(8.12)
HLbL LO	pheno: 10.3(9)		(5.69)
	lattice: 12.2(9)		(6.34)
HLbL NLO	pheno 0.26(6)		(5.70)
HVP LO	lattice: 713.2(6.1)		(3.37)
	pheno: tbc.		
HVP (N)NLO	pheno: -8.72(13)		(2.47)
total	11 659 203.3(6.2)		(9.4)

## QED [Aoyama, Hayakawa, Nio, Volkov, WP25.7]

- no significant changes since WP20
- five loop cross-checked by Volkov and bug found
- six-loop contribution  $\sim$  uncertainty due to parametric uncertainty of  $\alpha$ ,  $m_\ell$
- different values for  $\alpha$  (Cs, Rb,  $a_e$ ) differ by  $2.4\sigma$  to  $5.6\sigma$
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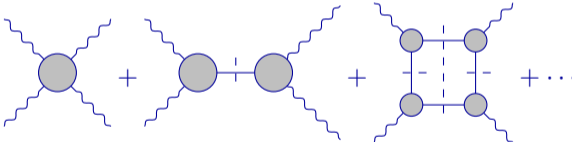
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## EW [Stöckinger, Stöckinger-Kim, WP25.8]

- LO  $\sim g^2/(16\pi^2)m_\mu^2/m_W^2 \sim 10^{-9}$
- calculated exactly at two-loop
- leading logarithm  $G_F\alpha^2 \log^2(M_Z^2/m_f^2)$  estimated
- leading uncertainty: EW HVP

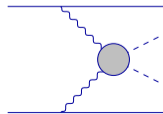
[Bijnens, Capiello, Eichmann, Fischer, González-Solís, Hermansson-Truedsson, Hoferichter, Hoid, Holz, Kubis, Kupść, Masjuan, Procura, Rebhan, Redmer, Rodríguez-Sánchez, Roig, Sánchez-Puertas, Stoffer, Zillinger, WP25.5]

- decompose  $a_{\mu}^{\text{HLbL}} \sim \int \langle 0 | j_{\text{em}}^{\mu}(x) j_{\text{em}}^{\nu}(y) j_{\text{em}}^{\rho}(z) j_{\text{em}}^{\sigma}(0) | 0 \rangle$
- multiple approaches: dispersive, holographic, ...
- dispersive: model hadrons in the blob

•  $a_{\mu}^{\text{HLbL}} \sim$  
 The diagram shows a series of Feynman diagrams representing the decomposition of the HLbL amplitude. It starts with a blob (grey circle) connected to four external wavy lines. This is followed by a plus sign, then a diagram with two blobs connected by a vertical line, with four external wavy lines. Another plus sign follows, then a diagram with four blobs arranged in a square, connected by horizontal and vertical lines, with four external wavy lines. The sequence ends with a plus sign and an ellipsis.

- needs data to be correct beyond pion pole

- e.g.  $ee \rightarrow ee\pi\pi$  and other resonances



[Feng, Gérardin, Jin, Lin, Meyer, WP25.6]

- use same decomposition, split by contributions (light quark, strange, charm, rest)
- additionally: relate to phenomenological calculations

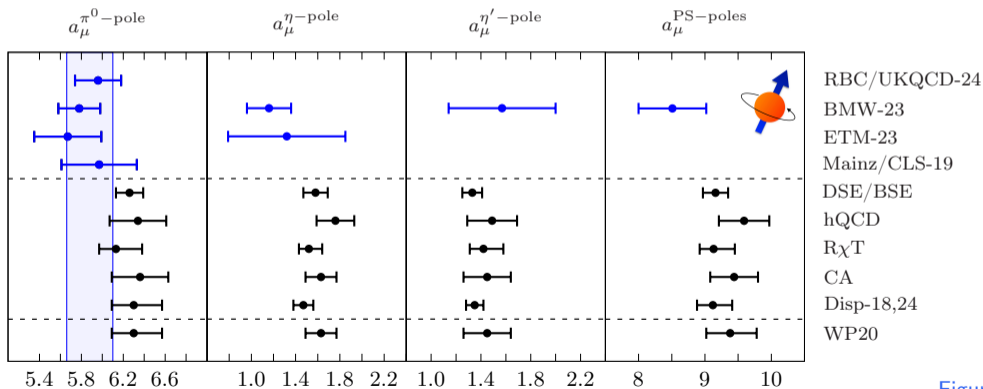
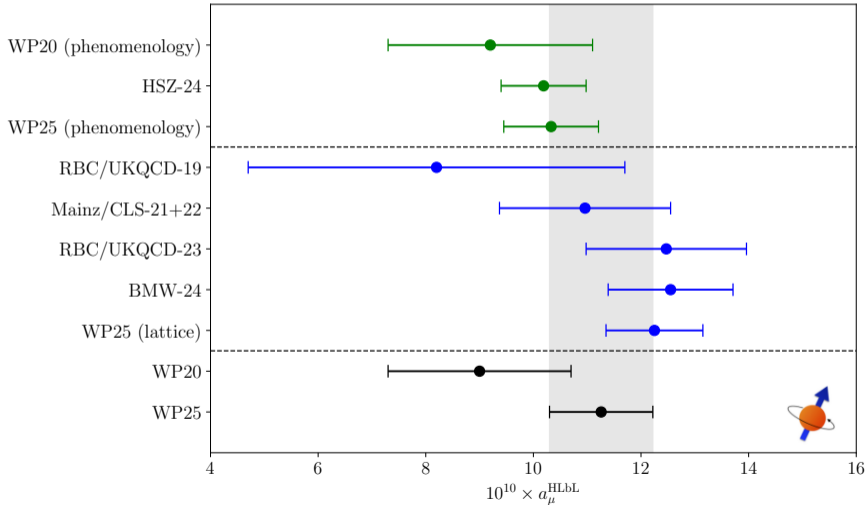


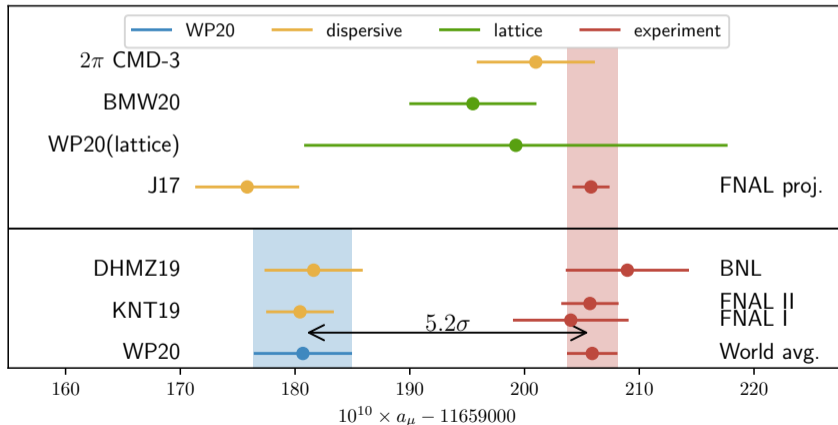
Figure 76

good agreement & consistency Figure 82



this is where all the trouble is..

- in WP20: combinations of  $e^+e^-$  data DHMZ & KNT in agreement, no good lattice
- shortly after WP20: good lattice (BMW), new  $2\pi$  data (CMD-3)



[Blum, Bruno, Cè, Clarke, Della Morte, DeTar, El-Khadra, Frezzotti, Gagliardi, Gérardin, Giusti, Gottlieb, Gülpers, Kuberski, Lahert, Lehner, Lellouch, Marinković, Meyer, Neil, Parrino, Portelli, Sitison, Tsang, Van de Water, Wang, Wittig, WP25.3]

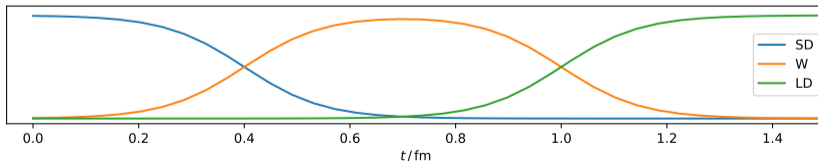
- calculation similar to HLbL:  $a_{\mu}^{\text{HVP}} \sim \int \langle 0 | j_{\text{em}}^{\mu}(x) j_{\text{em}}^{\nu}(0) | 0 \rangle$
- significant improvements since WP20: finite volume corrections, lattice scale uncertainty, full inclusion of IB, many more lattice actions are implemented
- 17 papers by 8 lattice collaborations, incl. 3 almost complete [BMW, RBC/UKQCD, Mainz]
- **everything is thoroughly checked!**
- introduction of window observables really helped!

## Section 3.6.1

$$\begin{aligned}
 a_\mu^{\text{HVP}} &= \underbrace{\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \text{diagram 5}}_{\sum_f a_\mu^{\text{HVP}}(f, \text{iso})} \\
 &= \sum_f a_\mu^{\text{SD}}(f, \text{iso}) + \sum_f a_\mu^{\text{W}}(f, \text{iso}) + \sum_f a_\mu^{\text{LD}}(f, \text{iso}) + \sum_f \delta a_\mu(\text{IB})
 \end{aligned}$$

Diagram 1: Quark loop with labels  $u, d$   
 Diagram 2: Quark loop with label  $s$   
 Diagram 3: Quark loop with label  $c$   
 Diagram 4: Two quark loops  
 Diagram 5: Quark loop with label  $m_u \neq m_d, \alpha \neq 0$

- split by contribution and short-distance, intermediate, long-distance window



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Diagram 1: Triangle with wavy line on top, two straight lines on sides, and a circle labeled 'u, d' at the bottom.

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Diagram 3: Triangle with wavy line on top, two straight lines on sides, and a circle labeled 'c' at the bottom.

Diagram 4: Triangle with wavy line on top, two straight lines on sides, and two circles at the bottom.

Diagram 5: Triangle with wavy line on top, two straight lines on sides, and a circle with a wavy line inside at the bottom.

Diagram 5 label:  $m_u \neq m_d, \alpha \neq 0$

- average all isospin-symmetric results for each window and add
- $a_\mu^{\text{LD}}(\text{iso})$  is tricky and was only computed by a few groups
- IB effects first added then averaged

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 &= \underbrace{69.06(22)}_{(3.21)} + \underbrace{236.16(42)}_{(3.24)} + \underbrace{407.9(5.0)}_{(3.28)} + \underbrace{0.1(3.4)}_{(3.30)}
 \end{aligned}$$

Diagram 1: Triangle with wavy line on top, quark loop labeled 'u, d'.  
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 Diagram 3: Triangle with wavy line on top, quark loop labeled 'c'.  
 Diagram 4: Triangle with wavy line on top, two quark loops.  
 Diagram 5: Triangle with wavy line on top, quark loop with a wavy line inside, labeled  $m_u \neq m_d, \alpha \neq 0$ .

- average all isospin-symmetric results for each window and add
- $a_\mu^{\text{LD}}(\text{iso})$  is tricky and was only computed by a few groups
- IB effects first added then averaged
- value and error dominated by LD, light quark channel

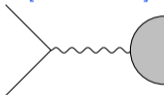
[Aliberti, Benton, Boito, Bruno, Carloni Calame, Cirigliano, Colangelo, Cotozzi, Cottini, Davier, Denig, Druzhinin, Golterman, Hoecker, Hoferichter, Hoid, Holz, Ignatov, Keshavarzi, Kubis, Kupich, Laporta, Leplumey, Liu, Logashenko, López Castro, Lutz, Malaescu, Maltman, Miranda, Müller, Nesterenko, Nomura, Passera, Peris, Piccinini, Pilato, Polat, Roig, Ruiz de Elvira, Signer, Stoffer, Teubner, Toledo, Ulrich, Venanzoni, Wright, Zaid, Zhang, WP25.2]

basic idea: relate HVP to data

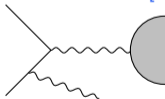
- $a_\mu^{\text{HVP}} = \text{diagram} \sim \int \frac{d^4q}{(2\pi)^4} \text{diagram}_{q^2}$
- dispersion relation  $\text{diagram}_{q^2} = \frac{1}{\pi} \int \frac{ds}{s - q^2} \Im(\text{diagram})$
- optical theorem  $2\Im(\text{diagram}) = \sum_{\text{had}} \int d\Phi |\text{diagram}|^2$
- measure either in  $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$  or  $\tau \rightarrow \nu_\tau + W(\rightarrow \text{hadrons})$  (+ flavour corrections)

- need to measure **all** hadronic final states as a function of  $s_h$
- kernel function favours low  $s_h$  contribution,  $> 70\%$  from  $\pi^+\pi^-$
- two ways of measuring:

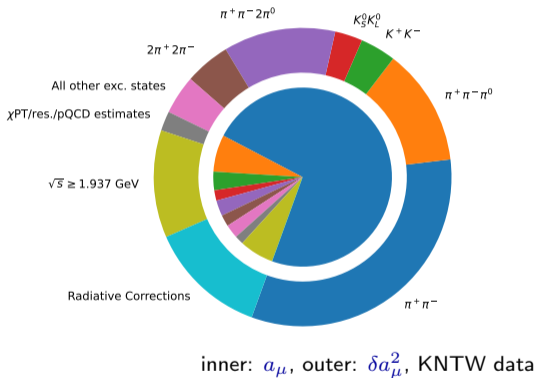
- scan [SND, CMD]



- radiative return [KLOE, BES, BaBar]



- three fitting groups: [KNTW, DHMZ, CHKLS]



## KNTW

- data dynamically lustered to prevent over-fitting
- utilises full given covariance matrices/assumes full systematic correlation
- fit to avoid incurred d'Agostini bias
- full covariance matrices propagated to final result

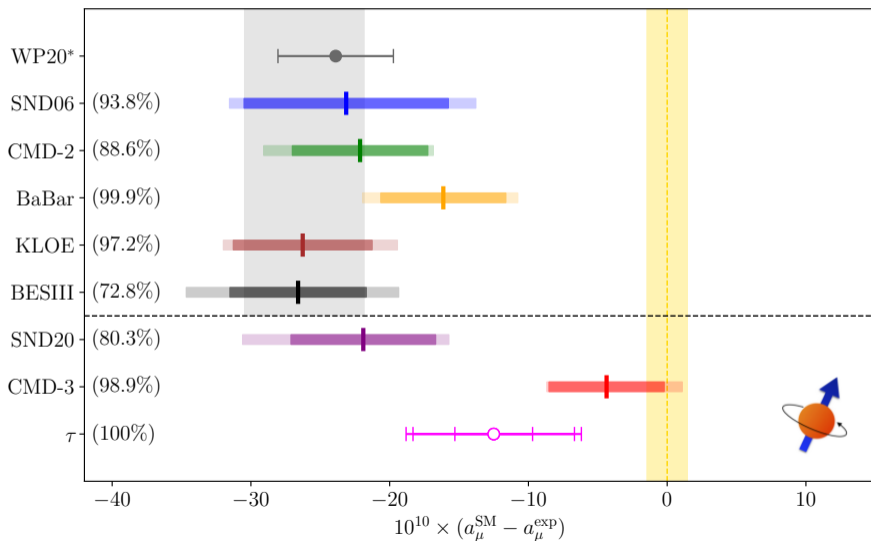
## DHMZ

- measurements quadratic spline interpolated and averaged on a fixed binning
- central value derived using uncertainties and local correlations
- uncertainties on channels generated from 'pseudo-experiments'

## CHKLS

- select channels' spectra constrained by analyticity and unitarity
- ⇒ fit functions
- consistent w. DHMZ, KNTW despite method differing significantly

old tensions between BaBar & KLOE, new tension between CMD-3 and everything else [Figure 27](#)





... we don't really know..

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⇒ for WP25, data-driven HVP was only used  
HVP-(N)NLO

- theory Monte Carlos, esp. for  $ee \rightarrow \pi\pi\gamma$  seem a bit sus..
- all rad. ret. measurements use PHOKHARA
- [DHMZ 23]: impact for KLOE & BES larger than for BaBar, refuted by [KLOE 24, BES 24]

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- all rad. ret. measurements use PHOKHARA
- [DHMZ 23]: impact for KLOE & BES larger than for BaBar, refuted by [KLOE 24, BES 24]
- RadioMonteCarLow2 Collaboration was founded to investigate this

### Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in $e^+e^-$ collisions

- Riccardo Aliberti<sup>1</sup>, • Paolo Beltrame<sup>2</sup>, • Ettore Budassi<sup>3,4</sup>, • Carlo M. Carloni Calame<sup>4</sup>, • Gilberto Colangelo<sup>5</sup>, • Lorenzo Cotrozzi<sup>2</sup>, • Achim Denig<sup>1</sup>, • Anna Driutti<sup>6,7</sup>, • Tim Engel<sup>8</sup>, • Lois Flower<sup>2,9</sup>, • Andrea Gurgone<sup>3,6,7</sup>, • Martin Hoferichter<sup>5</sup>, • Fedor Ignatov<sup>2</sup>, • Sophie Kollatzsch<sup>10,11</sup>, • Bastian Kubis<sup>12</sup>, • Andrzej Kupś<sup>13,14\*</sup>, • Fabian Lange<sup>10,11</sup>, • Alberto Lusiani<sup>7,15</sup>, • Stefan E. Müller<sup>16</sup>, • Jérémy Paltrinieri<sup>2</sup>, • Pau Petit Rosàs<sup>2</sup>, • Fulvio Piccinini<sup>4</sup>, • Alan Price<sup>17</sup>, • Lorenzo Punzi<sup>7,15</sup>, • Marco Rocco<sup>10,16</sup>, • Olga Shekhovtsova<sup>19,20</sup>, • Andrzej Siódmok<sup>17</sup>, • Adrian Signer<sup>10,11\*</sup>, • Giovanni Stagnitto<sup>21</sup>, • Peter Stoffer<sup>10,11</sup>, • Thomas Teubner<sup>2</sup>, • William J. Torres Bobadilla<sup>2</sup>, • Francesco P. Ucci<sup>3,4</sup>, • Yannick Ulrich<sup>2,5\*</sup> and • Graziano Venanzoni<sup>2,7\*</sup>  
(RadioMonteCarLow 2 working group)

#### Abstract

We present the results of Phase I of an ongoing review of Monte Carlo tools relevant for low-energy hadronic cross sections. This includes a detailed comparison of Monte Carlo codes for electron-positron scattering into a muon pair, pion pair, and electron pair, for scan and radiative-return experiments. After discussing the various approaches that are used and effects that are included, we show differential cross sections obtained with AFKQED, BABA YAGA@NLO, KKMC, MCGPJ, MCMULE, PHOKHARA, and SHERPA, for scenarios that are inspired by experiments providing input for the dispersive evaluation of the hadronic vacuum polarisation.

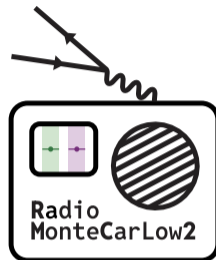
[RMCL2 23, WP25.2.4]

## Phase 1 [radiomontecarlow2.gitlab.io](https://radiomontecarlow2.gitlab.io)

- seven codes: AFKQED, BABAYAGA@NLO, KKMC, MCGPJ, McMULE, PHOKHARA, SHERPA
- calculate  $ee \rightarrow XX(\gamma)$  with  $X \in \{e, \mu, \pi\}$
- five scenarios: CMD-like, KLOE-like SA, KLOE-like LA, BES-like, B-like
- all calculate different things, compare to see what matters fixed order vs. YFS vs. collinear resummation vs. parton shower, ...

## goals

- keep vital Monte Carlo software alive & accessible
- support development by pooling resources
- focus on low energy
- get people to stop calling it ISR



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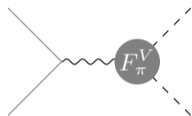
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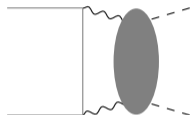
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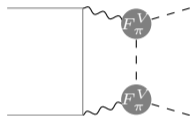
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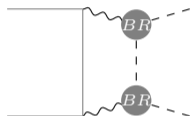
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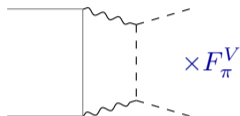
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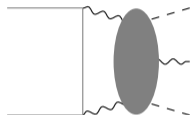


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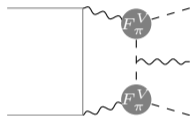


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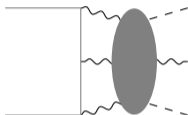
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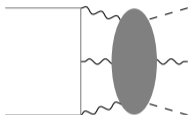
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- $F \times$ sQED is known to be insufficient, at least for  $ee \rightarrow \pi\pi$  [Colangelo, Hoferichter, Monnard, de Elvira 22, Ignatov, Lee 22]
- effect for  $ee \rightarrow \pi\pi\gamma$  maybe not as bad as feared [Petit Rosàs, Shekhovtsova, Torres Bobadilla 26]

- fixed-order: calculate all diagrams exactly\* at LO/NLO/NNLO/... in  $\alpha$   
(some approximations like  $m_e^2 \ll m_\mu^2, s, \dots$  may be necessary)
- note that  $2 \rightarrow 3$  (N)NLO  $\subset$   $2 \rightarrow 2$  (N)NNLO
- resummation: approximate higher-order corrections rather than calculate
- YFS / CEEEX / Photos: capture soft photons, potentially with interference
- PS / CS: capture collinear photons numerically or semi-analytically
- ISC / FSC: corrections only from initial state or final state, for  $ee \rightarrow \pi\pi(\gamma)$  this means only  $n = 1$
- event generator vs. integrator

in Phase I for Scan or Rad. return

AFKQED **R**: LO+ISC with CS, FSC with Photos,  $X \in \{\mu, \pi\}$

BABAYAGA **S**: NLO+PS, **R**: LO+PS; F $\times$ sQED

KKMC **S**: CEEEX, **R**: CEEEX for  $X = \mu$

MCGPJ **S**: NLO+CS, **R**: LO+CS for  $X \in \{e, \mu\}$ ; GVMD

McMULE **S**: NNLO, **R**: NLO; for  $X = \pi$  only ISC

PHOKHARA **R**: NLO for  $X \in \{\mu, \pi\}$ , F $\times$ sQED

SHERPA **S**: NLO+YFS for  $X \in \{e, \mu\}$ , YFS sQED for  $X = \pi$

in Phase I for Scan or Rad. return

⇒ Phase II (might be out of date!)

AFKQED **R**: LO+ISC with CS, FSC with Photos,  $X \in \{\mu, \pi\}$

BABAYAGA **S**: NLO+PS, **R**: LO+PS; F×sQED  
⇒ FsQED/GVMD, **R**: NLO+PS

KKMC **S**: CEEEX, **R**: CEEEX for  $X = \mu$

MCGPJ **S**: NLO+CS, **R**: LO+CS for  $X \in \{e, \mu\}$ ; GVMD

McMULE **S**: NNLO, **R**: NLO; for  $X = \pi$  only ISC  
⇒ **S**: FsQED, **R**: partial NNLO, Pisa consensus WIP

PHOKHARA **R**: NLO for  $X \in \{\mu, \pi\}$ , F×sQED  
⇒ GVMD done, NNLO & CEEEX WIP

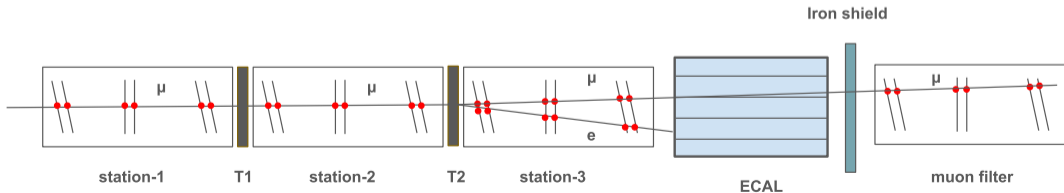
SHERPA **S**: NLO+YFS for  $X \in \{e, \mu\}$ , YFS sQED for  $X = \pi$   
⇒ GVMD & NNLO

- $\tau$  data: measure  $\tau \rightarrow \nu_\tau + W(\rightarrow \pi^0\pi)$  [WP25.2.7]
  - in isospin-symmetric theory:  $W$  couples to hadrons like a photon
  - isospin breaking corrections need to be calculated
- ⇒ not included in WP25 due to potentially large unknown uncertainties.
  - value in the middle of  $e^+e^-$  and lattice

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- lattice hybrids: combine data and lattice [BWM-DMZ 24, WP25.4]
  - use data where it's (relatively) consistent ( $\geq 2.8\text{fm}$ ) but lattice error is large
  - results in very good error ( $3.8 \times 10^{-10}$  vs. WP25  $6.2 \times 10^{-10}$ )
  - **however** highly contentious and **NOT** included in WP25
    - does not address tension
    - highly dependent on switchover
    - KNTW vs. CMD-3 is still  $2.8\sigma$

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- MUonE: measure HVP in  $\mu$ - $e$  scattering (cf. Antonio's discussion, [WP25.2.10])

- scattering 160 GeV muon beam on low- $Z$  material ( ${}_4\text{Be}$ )
  - pure  $t$ -channel  $-s \simeq Q^2 \simeq 0$
- ⇒ high  $s \leftrightarrow$  measure more of the curve
- $10^{-2}$  measurement  $\times 10^{-3}$  effect =  $10^{-5}$  precision
  - NNLO calculated, effect  $10^{-3} - 10^{-4}$ , depending on scenario [Broggio, Engel, Ferrogli, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]
- ⇒ partial N<sup>3</sup>LO and resummation needed



## clarification from last week

- WP25 uses lattice for HVP
- data **is** used but only for HVP at (N)NLO because lattice is not ready yet
- very conservative estimate: envelope of KNT and KNT+CMD3
- $\delta a_\mu \lesssim \text{HVP (N)NLO}$  so even if you don't believe the data at all, it's fine

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## conclusion & opinions

- $(g - 2)_\mu$  is not a solved problem
- is the explanation BSM? probably not..
- lattice works, thanks to window observables
- lattice vs. data needs to be solved!
- 'Resolving the puzzles will require new measurements, together with an improved understanding of higher-order radiative corrections and their implementation in MC generators.'

Figure 40

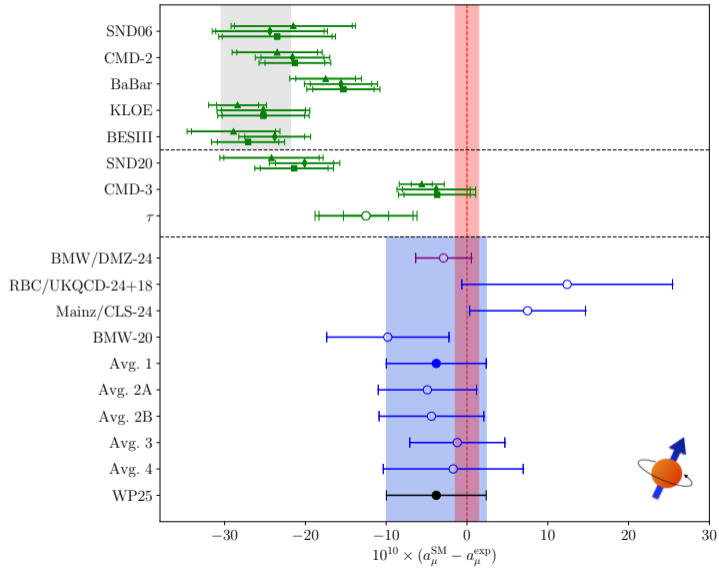


Figure 38

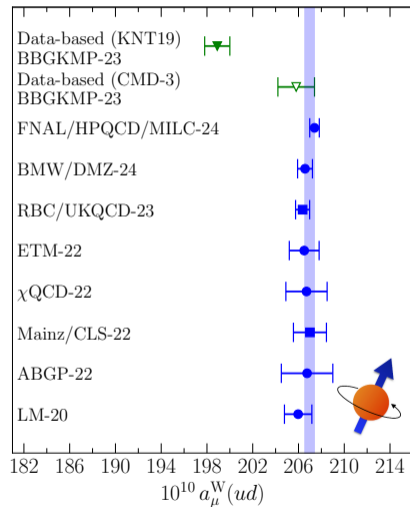
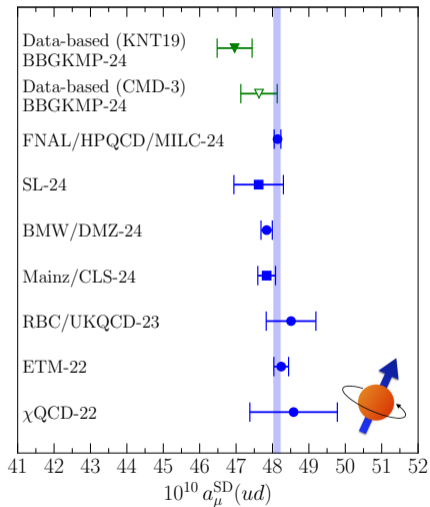


Figure 38

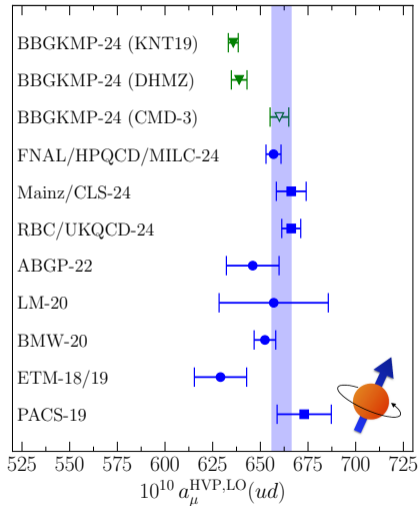
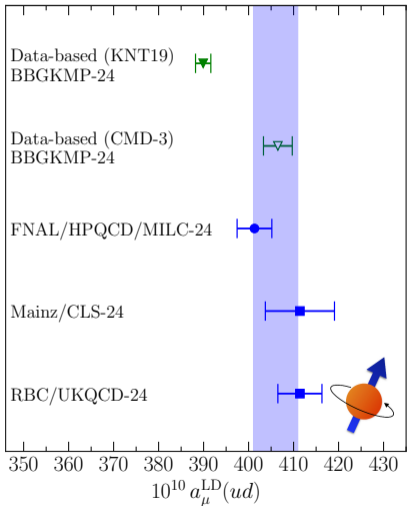


Figure 2

