Hadron Physics 2030

Radiative corrections to ℓ-N scattering with McMule

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fixed-order NNLO QED framework Monte Carlo for MUons and other LEptons

- provided: matrix elements by us or others
- output: physical cross section for any physical observable
- McMule: phase space generation, subtraction, stabilisation, integration, event generation, etc.

- all leptonic $2 \to 2$ processes in QED at NNLO ($+$ a few others)
- stable public version is an integrator
- generator on development branch

Get the code here: `https://mule-tools.gitlab.io`
Read the docs here: `https://mcmule.readthedocs.io`

McMule

processes in McMule

process	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$\ell p \to \ell p$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO
$eN \to eN$	PRad, ULQ2	background	NNLO−
$e^-e^- \to e^-e^-$	Prad 2	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \ell\ell$	VEPP, BES, Daphne, ...	R-ratio	NNLO+
	Belle	τ properties	
$ee \to \gamma\gamma$	Daphne	dark searches	NNLO−
	any e^+e^- collider	luminosity measurement	
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO−
$\mu \to \nu\bar{\nu}e$	MEG	ALP searches	NNLO+
	DUNE	beam-line profiling	
$\mu \to \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \to \nu\bar{\nu}eee$	MEG, Mu3e	background	NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	NLO+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	NLO+

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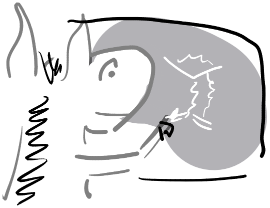
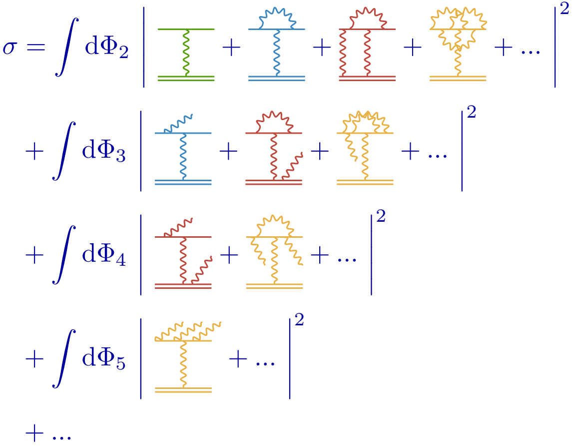
$e^-e^- \to e^-e^-$	Prad 2	on	NNLO
	MOLLER, ...	low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- col	measurement	NNLO
$ee \to \ell\ell$	VEPP, BES,		NNLO+
	Belle	s	
$ee \to \gamma\gamma$	Daphne	es	NNLO−
	any e^+e^- col	measurement	
$e\nu \to e\nu$	DUNE	θ_W	NNLO−
$\mu \to \nu\bar{\nu}e$	MEG		NNLO+
	DUNE	filing	
$\mu \to \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \to \nu\bar{\nu}eee$	MEG, Mu3e	background	NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	NLO+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	NLO+

goal: world domination



theory background

anatomy of an NNNLO calculation



challenges to overcome

- divergent phase space integration

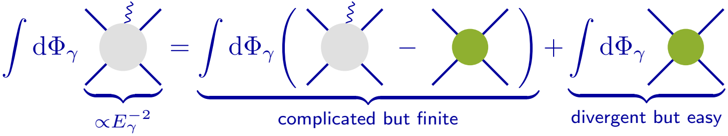
⇒ FKS$^\ell$

- numerical instabilities

⇒ next-to-soft stabilisation

- virtual amplitudes with $m \neq 0$

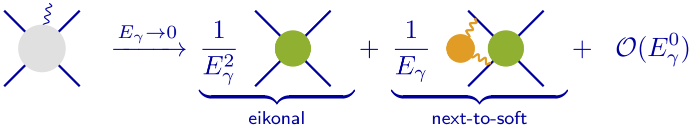
⇒ OpenLoops (one-loop) massification (two-loop) subtract universal counter term from divergent real correction



- works to all order in QED [Engel, Signer, YU 19]
- no resolution parameter ω_c
- unphysical & arbitrary $0 < \xi_c \lesssim 1$
- singularities are treated locally $\to$ stable numerical integration

next-to-soft stabilisation

real-virtual corrections trivial in principle, delicate in practise

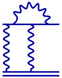


- based on LBK theorem [Low 58; Burnett, Kroll 67] and extensions [Engel, Signer, YU 21; Engel 23; Engel 24]
- if $E_\gamma < E_{\text{NTS}} \approx 10^{-3}\sqrt{s}/2$, switch to NTS expansion rather than full expression
- introduces small theory error $\mathcal{O}(10^{-3}) \times \sigma^{(2)} = \mathcal{O}(10^{-6})$
  $\Rightarrow$ well below the N^{3}LO
- significant speed-up: 7 days vs. 3 months

massification

two-loop integrals with masses are really difficult

- but $m_\ell^2 \ll m_p^2 \sim s \sim Q^2$
- expand in m_ℓ^2/Q^2



$$\sim A \log^2 \frac{m_\ell^2}{Q^2} + B \log \frac{m_\ell^2}{Q^2} + C + \mathcal{O}\Big(\frac{m_\ell^2}{Q^2}\Big)$$

- can be done easily by using $m_\ell = 0$ result up to three-loop
  [Penin 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18; YU 23]
- introduces small theory error $\mathcal{O}(10^{-2}) \times \sigma^{(2)} = \mathcal{O}(10^{-5})$
  $\Rightarrow$ well below statistical error

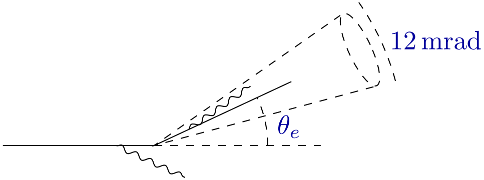
results for PRad

setup for $ep \to ep$ and $ee \to ee$

$E_{\text{beam}} = 1.1\,\text{GeV}$

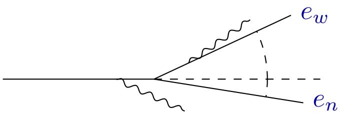
e-p:

- $0.7^\circ < \theta_e < 6.0^\circ$
- no more than $20\,\text{MeV}$ photons outside a $12\,\text{mrad}$ cone around the electron



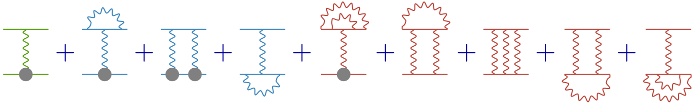
e-e:

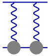
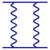
- $0.5^\circ < \theta_e < 6.5^\circ$
- $\sum E_\gamma < 131.95\,\text{MeV}$, $\big|180^\circ - |\phi_n - \phi_w|\big| < 7.35^\circ$

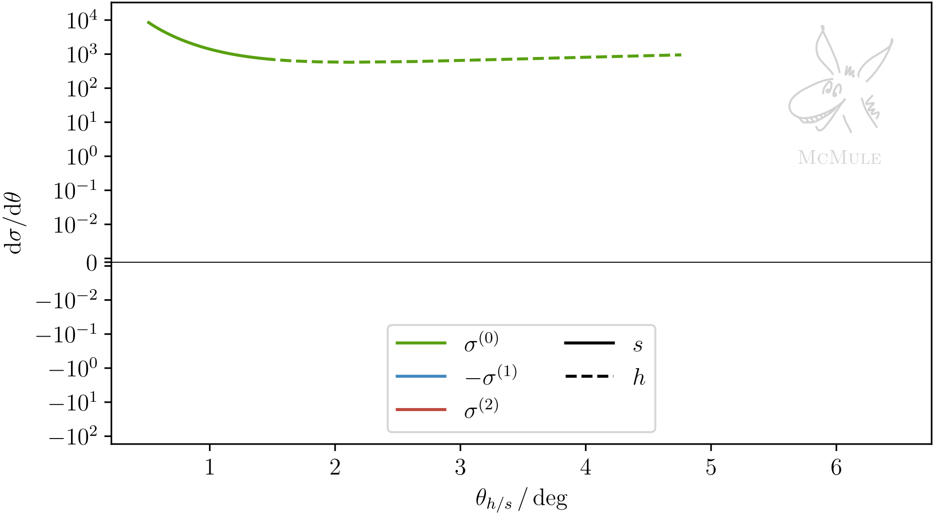


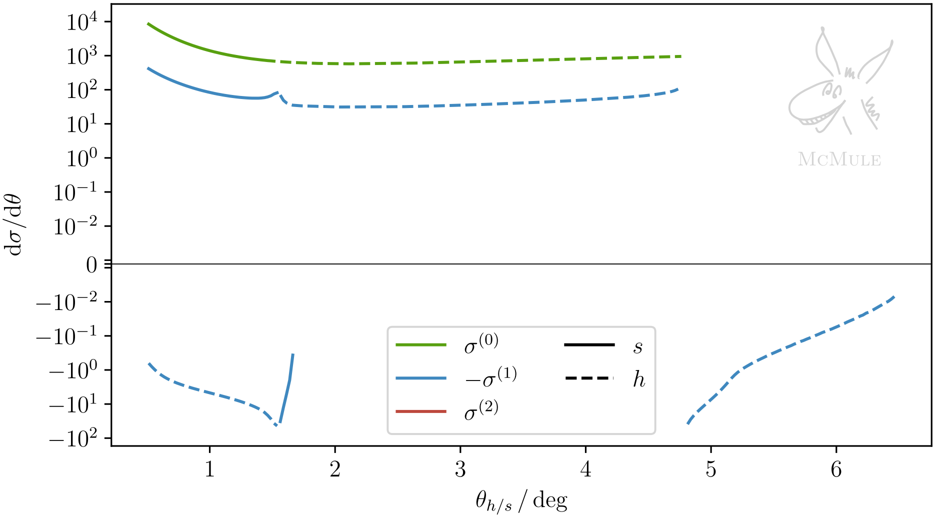
not to scale

- simple dipole model for proton form factor $G_E = \frac{G_M}{1+\kappa} = \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2}$

$\sigma \supset$ 

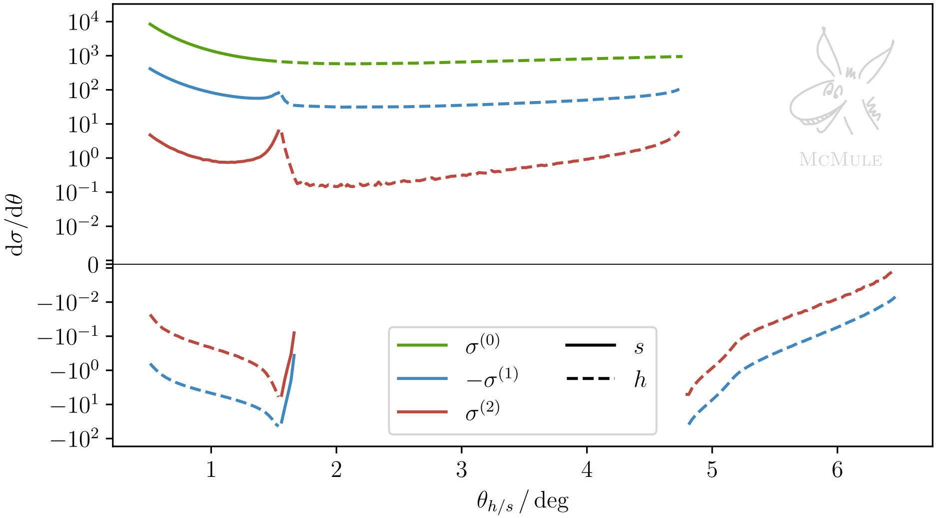
- study TPE effects with $\Delta\mathrm{TPE} =$  $(\Lambda^2) -$  $\Big|_{\Lambda^2=\{0.88,0.71,0.60\}\,\mathrm{GeV}^2}$


10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0
-10^{-2}
-10^{-1}
-10^0
-10^1
-10^2
$d\sigma/d\theta$
1
2
3
4
5
6
$\theta_{h/s}$ / deg
$\sigma^{(0)}$
$-\sigma^{(1)}$
$\sigma^{(2)}$
s
h
McMule

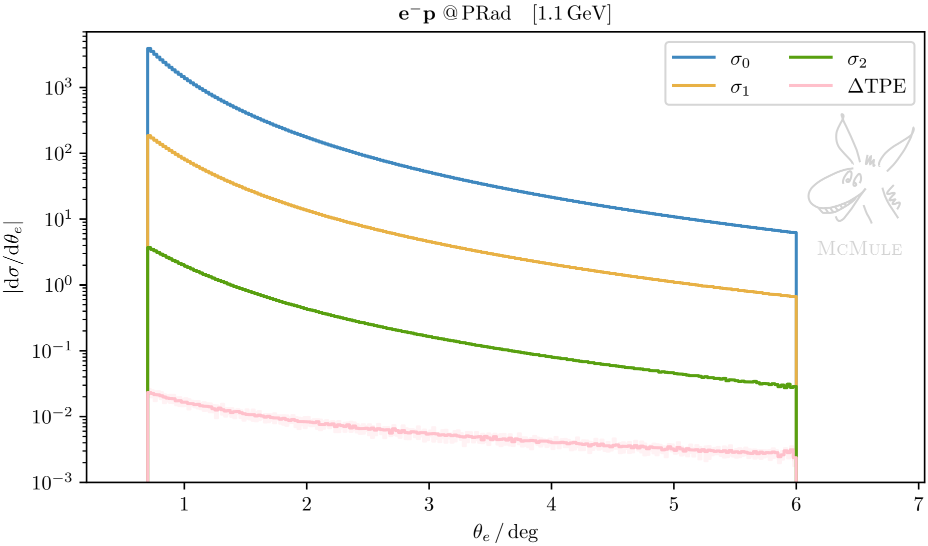


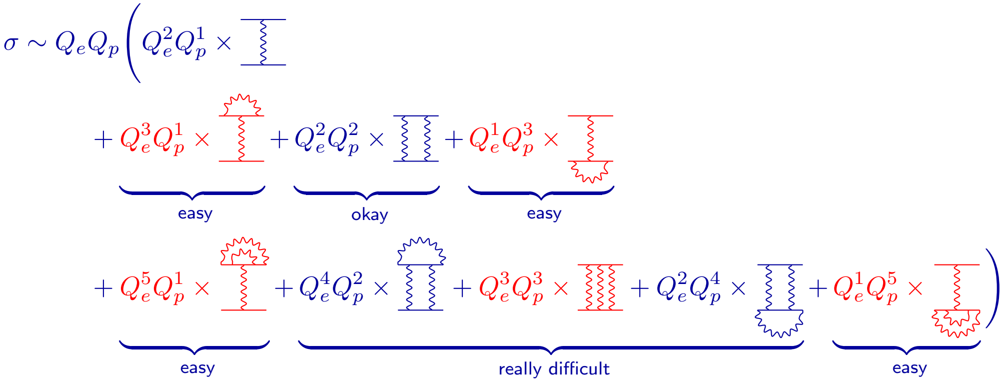
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0
-10^{-2}
-10^{-1}
-10^0
-10^1
-10^2
$d\sigma/d\theta$
1
2
3
4
5
6
$\theta_{h/s}$ / deg
$\sigma^{(0)}$
$-\sigma^{(1)}$
$\sigma^{(2)}$
s
h
McMule

results for e-e scattering

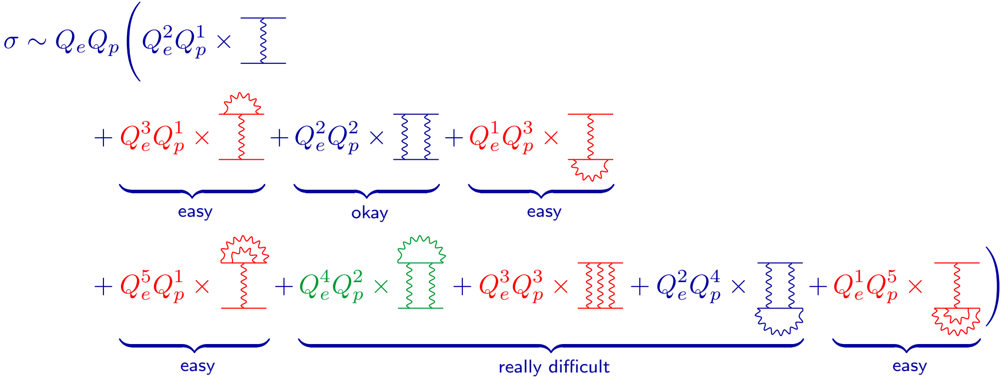


results for e-p scattering



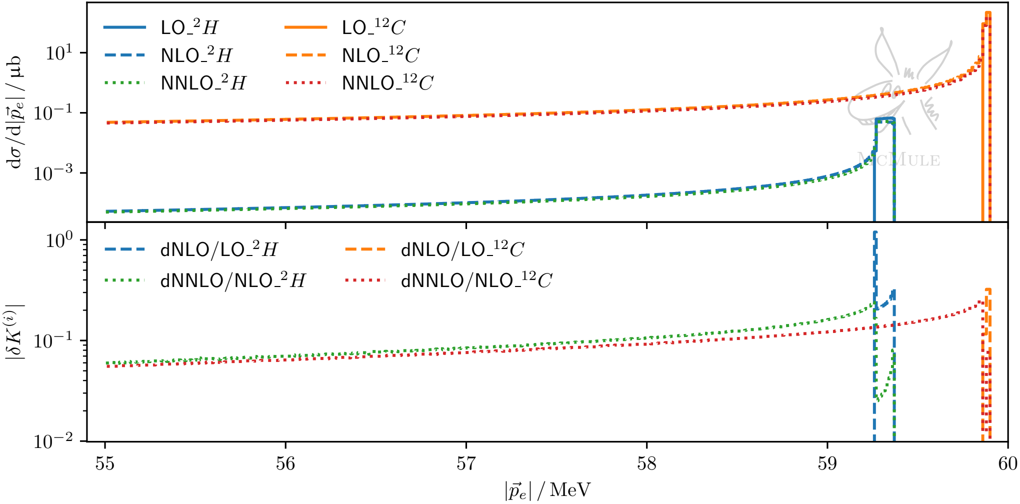


- by using e^+ and e^- beams you can make the red or blue stuff go away
  - $\sigma_{e^+} + \sigma_{e^-} \ \rightarrow$ some of the theoretically difficult stuff cancels
  - $\sigma_{e^+} - \sigma_{e^-} \ \rightarrow$ radiative corrections are reduced



- by using e^+ and e^- beams you can make the red or blue stuff go away
  - $\sigma_{e^+} + \sigma_{e^-} \to$ some of the theoretically difficult stuff cancels
  - $\sigma_{e^+} - \sigma_{e^-} \to$ radiative corrections are reduced
- there may be hope for this

ULQ2: $e\{^2H,^{12}C\} \to e\{^2H,^{12}C\}$ ($E_e = 60$ MeV, $48^\circ < \theta_e < 52^\circ$, $|\vec{p}_e| > 55$ MeV)

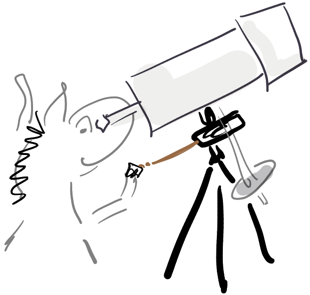


point-like calculations

- approximate NNLO for $ep \to ep\gamma$ from $pp \to 2j + \gamma$ [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia 23]
- full mass dependence plausible but very difficult

form-factor calculations

- partial N^{3}LO for $ep \to ep$ (and who knows, maybe $ee \to ee$)
- various performance & usability improvements
- arbitrary spin nucleus (is there any use case?) using [Lorcé 09]

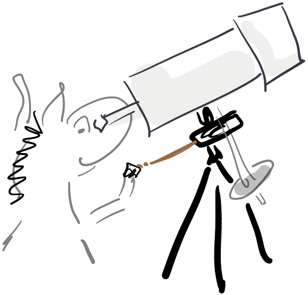


better hadronic models

- TPE: more flexible & better models, maybe extension to some other nuclei
- leptonic QED for inelastic: is there interest?

general

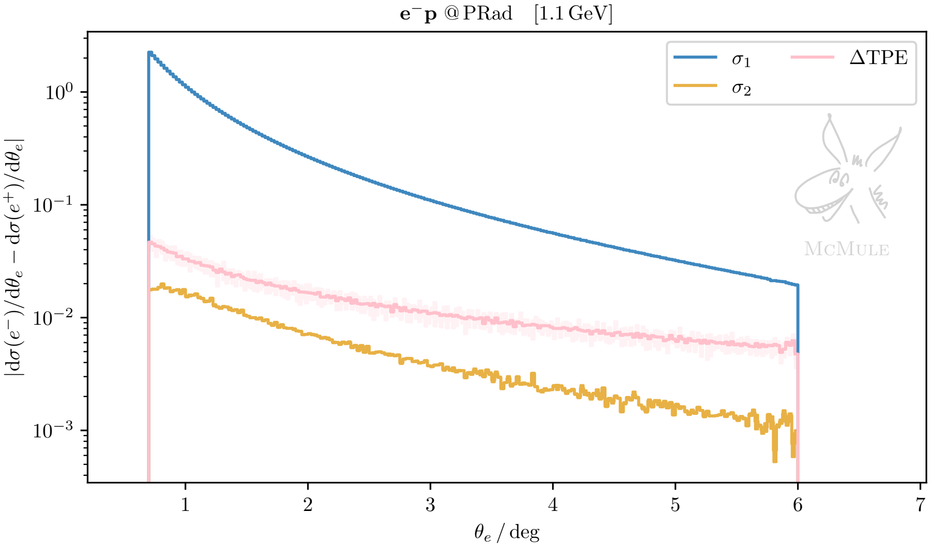
- merging the generator
- resummation of soft photon emission
- EW & polarisation effects for $ee \to ee$, $ep \to ep$, ...



f.l.t.r.: S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), V.Sharkovska (Zurich & PSI), S.Gündogdu (Zurich & PSI), D. Moreno (PSI), A.Coutinho (IFIC), Y.Ulrich (Liverpool), D. Radic (Zurich & PSI), L.Naterop (Zurich & PSI), M.Rocco (Turin)
not shown: F.Hagelstein (Mainz), N.Schalch (Oxford), T.Engel (Freiburg), A.Gurgone (Pavia), P.Banerjee (Cosenza)

McMule

`mule-tools.gitlab.io`



e⁻p @ PRad [1.1 GeV]
σ1
σ2
ΔTPE
McMule
10^0
10^{-1}
10^{-2}
10^{-3}
$|d\sigma(e^-)/d\theta_e - d\sigma(e^+)/d\theta_e|$
1
2
3
4
5
6
7
θ_e / deg

TPE subtractions