McMule

UNIVERSITY OF LIVERPOOL

TP Seminar

High precision QED with McMule

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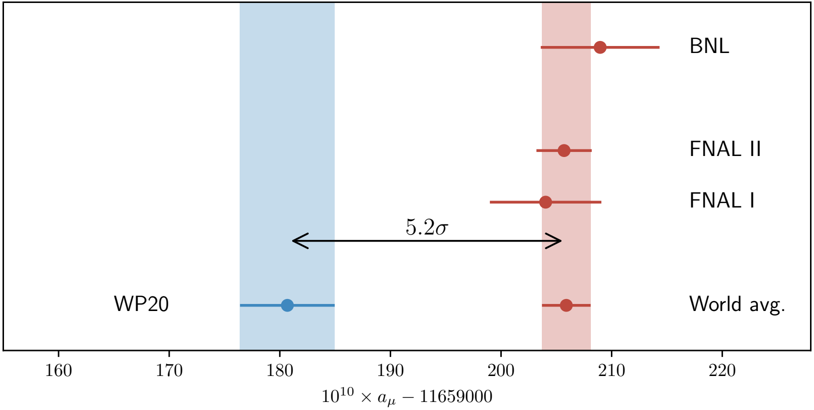
University of Liverppol

16 October 2024

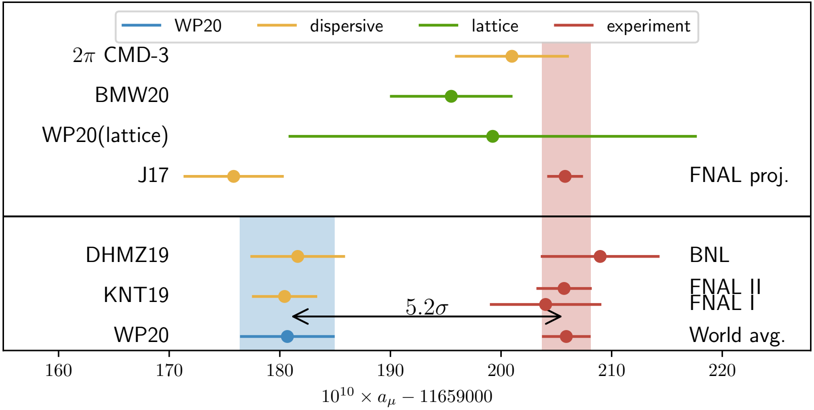
reminder: the $g-2$ situation

it used to be very clean...

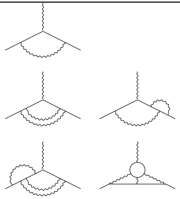
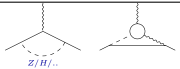
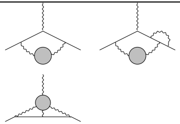


reminder: the $g-2$ situation

it used to be very clean... now, less so..

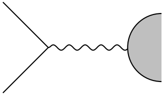


$g-2$ theory

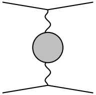
	value	diagrams
QED 1-loop	$\alpha/2\pi = 11\,614\,097.3$	
QED 2-loop	$-17\,723.1$	+ 3 others + 1 conspiracy theory
QED 3-loop	148.0	+ 70 others
more QED	-0.5	
EW	15.3	
HVP	$684.5(4.0)$	 + others
HLbL	$9.2(1.7)$	
total	$11\,659\,181.0(4.3)$	[$g-2$ white paper 20]
FNAL+BNL	$11\,659\,206.2(4.0)$	

the HVP is where all the trouble is

time-like in $ee \to$ hadrons

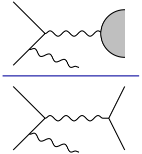
$\int \mathrm{d}s \Big(K(s)$  $\Big)$

space-like in $e\mu \to e\mu$

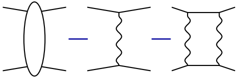
$\int \mathrm{d}t \Big(K'(t)$  $\Big)$

... but what actually happens ...

radiative return measurement

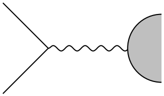
$\int \mathrm{d}s \Big(K(s)$  $\Big)$

loop-induced process

$\int \mathrm{d}t\, K'(t) \Big($  $\Big)$

radiative corrections are vital

time-like in $ee \to$ hadrons

$\int \mathrm{d}s \Big(K(s)$  $\Big)$

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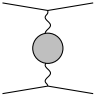
$\int \mathrm{d}t \Big(K'(t)$  $\Big)$

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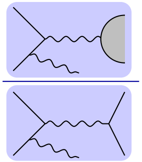
radiative return measurement

$\int \mathrm{d}s \Big(K(s)$  $\Big)$

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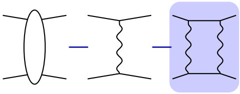
$\int \mathrm{d}t \, K'(t) \Big($  $\Big)$

radiative corrections are vital

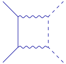
benefiting from LHC technology where possible

- soft resummation: CEEX ($\to$ improved YFS exponentiation)
- collinear resummation: parton shower & structure functions
- $2 \to 2$ with mass dependence at NNLO

  $\Rightarrow$ precision: $\mathcal{O}(10^{-4})$

  MUonE needs 10^{-5}
- $2 \to 3$ with mass dependence at NLO

  $\Rightarrow$ precision: $\mathcal{O}(\text{few} \times 10^{-3})$

  radiative return needs NNLO for kinematics
- pion final states: often only very simplified models $\left|F_\pi(s)\right|^2 \times$ 

  full hadronic model needed

just like @ the LHC ...

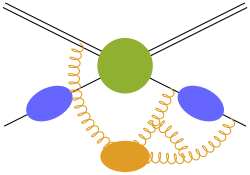
amplitude ⟶ implementation ⟶ cross section

just like @ the LHC ...

... except fermion masses are physical $\Rightarrow$ need massive amplitude

- they are also small $\rightarrow$ we can drop terms $\sim \left(\frac{\alpha}{\pi}\right)^2 \log\frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$ if $m^2 \ll Q^2$
- based on SCET factorisation & method of regions [Penin 06; Mitov, Moch 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g. $e\mu \to e\mu$ at two-loop:

$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$



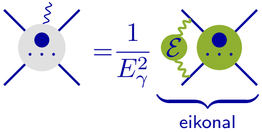
- soft: process-dependent $S = 1 +$ fermion loops
  $\rightarrow$ compute separately to combine with hadron loops
- collinear: universal Z, converts $1/\epsilon \to \log(m^2/Q^2)$

amplitude $\longrightarrow$ implementation $\longrightarrow$ cross section

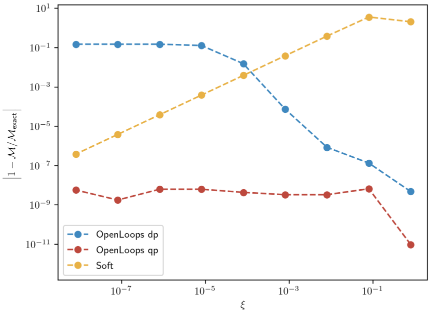
how does an NNLO QED calculation work?

just like @ the LHC ...

... except the real-virtual can be delicate b/c it's more exclusive



$$+\mathcal{O}\left(E_\gamma^{-1}\right)$$



amplitude ⟶ **implementation** ⟶ cross section

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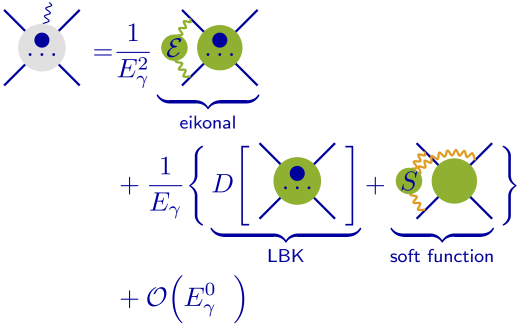
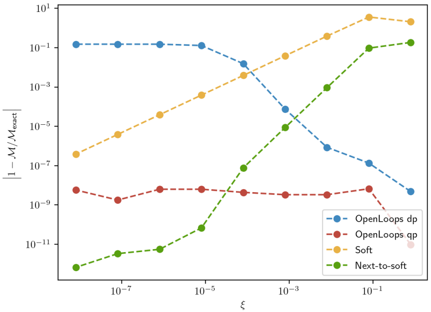
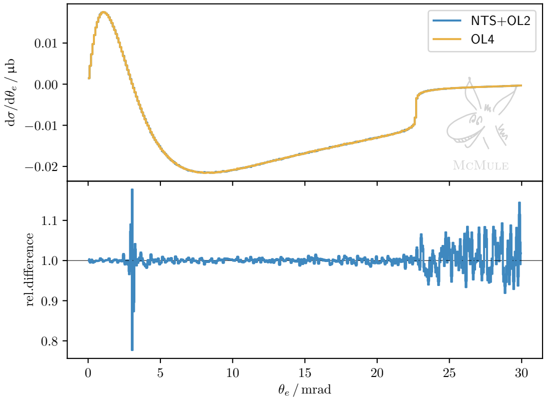




amplitude ⟶ implementation ⟶ cross section

test next-to-soft stabilisation vs OL4 (OpenLoops quad) for $\mu e \to \mu e$ real-virtual



- same statistics, same result
- 70 days vs 4 days
- integrated results for different cuts

⇒ this is not an approximation but a numerical tool

NTS	OL4
$-0.29268(4)$	$-0.29267(4)$
$-0.44789(6)$	$-0.44778(6)$
$-0.64662(9)$	$-0.64649(9)$

beyond real-virtual

this works for any number of photons and loops [Engel 23, 24] (this includes an elegant proof of YFS)

- proof using HQET and lots of combinatorics to show that
  - hard region: LBK [Low 58; Burnett, Kroll 67] + magnetic insertion
  - soft region: one-loop exact
  - two-photon function: tree-level exact
  - unordered multi-photon emission reduces to one- or two-photon case
- magnetic insertion vanishes for $\sum |\mathcal{A}|^2$, polarised case possible to three loops [Grozin, Marquard, Piclum, Steinhauser 07]
- automated construction of LBK term through shifts $\{p_i, n_i\} \to \{p_i + \delta p_i, n_i + \delta n_i\}$ [Bonocore, Kulesza 21; Balsach, Bonocore, Kulesza 23]

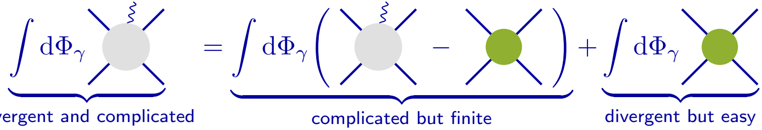
the whole process is fully automatised and requires no more input than eikonal!

how does an NNLO QED calculation work?

just like @ the LHC ...

- universal soft limit $\mathcal{M}^{(\ell)}_{n+1} = \mathcal{E}\mathcal{M}^{(\ell)}_{n} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}^{(\ell)}_{n} = \sum_{\ell=0}^{\infty} \mathcal{M}^{(\ell)f}_{n} =$ finite

use this to construct an all-order subtraction scheme FKS$^\ell$ [Engel, Signer, YU 19]



amplitude ⟶ implementation ⟶ cross section

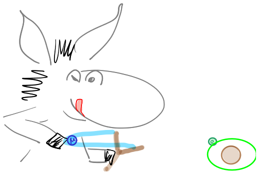
- scattering μ of low-Z material $({}_4\mathrm{Be})$
- pure t-channel $-s \simeq Q^2 \simeq 0$

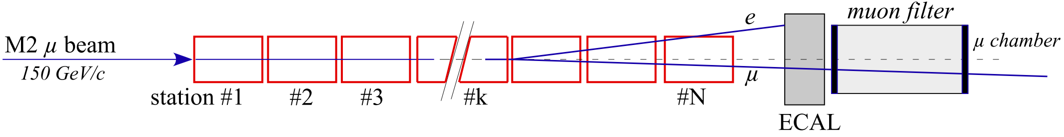
$\Rightarrow$ high $s \leftrightarrow$ measure more of the curve

- beam energy needs to be quite high $E_\mu \simeq 160\,\mathrm{GeV}$

$\Rightarrow$ M2 muon beam at CERN North Area

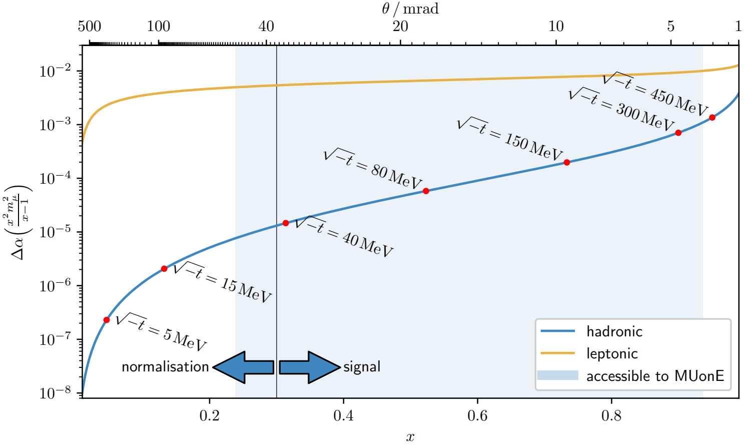
- main measurement: θ_e, θ_μ
  - $+$ E_{beam} for calibration
  - $+$ E_μ for particle ID





signal & normalisation regions

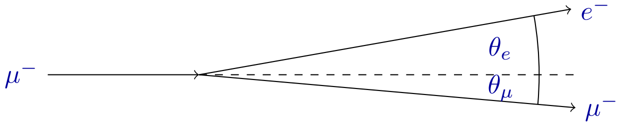
cancel systematic effects $\left(\mathrm{d}\sigma/\mathrm{d}\theta\right)_{\text{sig}} \Big/ \left(\mathrm{d}\sigma/\mathrm{d}\theta\right)_{\text{norm}}$



implemented in McMule v0.4.2

https://mule-tools.gitlab.io

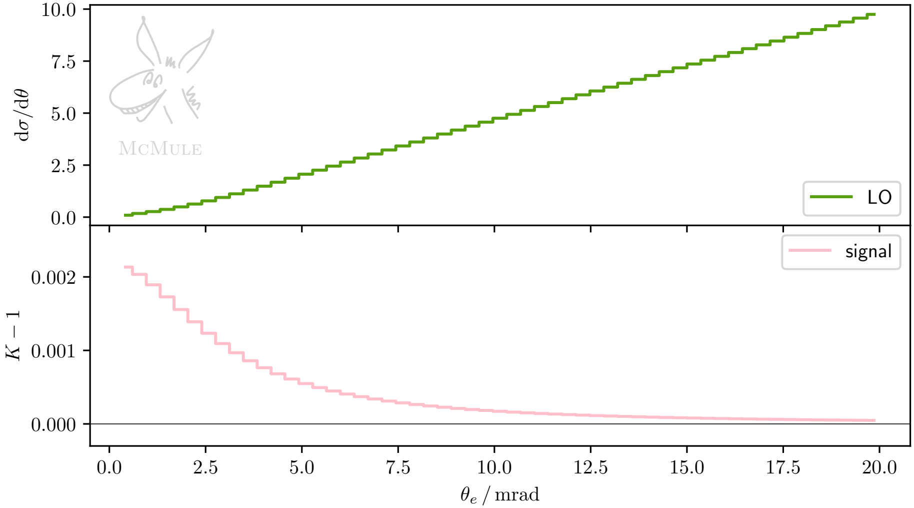
- $\mu^- e^- \to \mu^- e^-$



- S1: $E_e > 1\,\mathrm{GeV}$, $\theta_\mu > 0.3\,\mathrm{mrad}$
- run for 2.5 CPU yr
  (290 kWh energy / 3.5 kgCO2e)

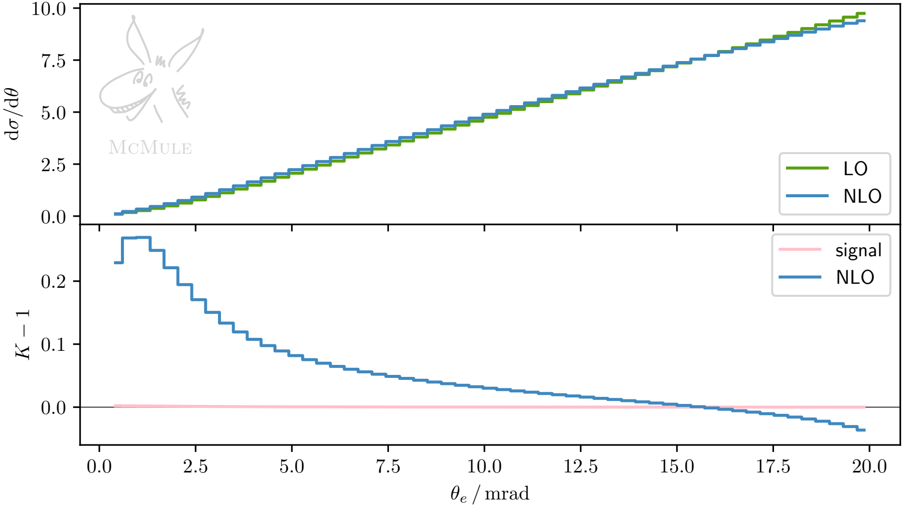
[Broggio, Engel, Ferroglia, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]

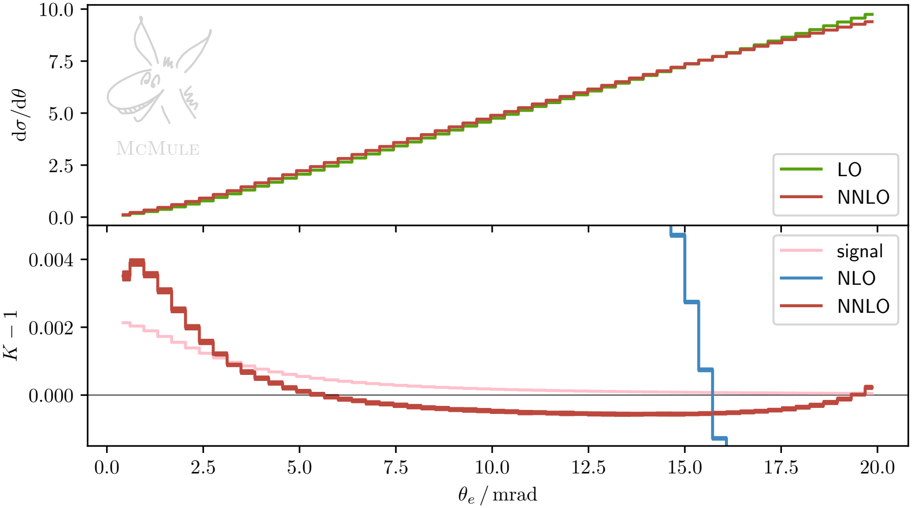
all results and data: https://mule-tools.gitlab.io/user-library/mu-e-scattering/muone-full-legacy/



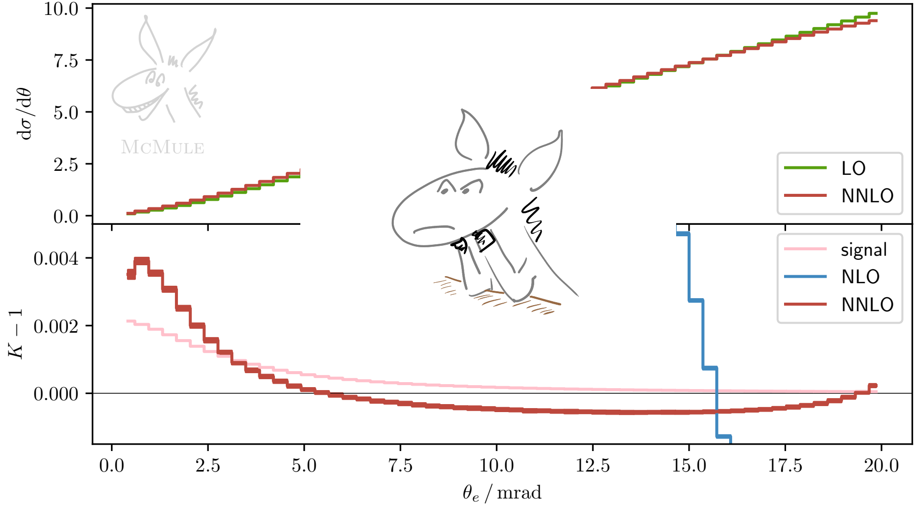
10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
LO
signal
0.002
0.001
0.000
K − 1
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θe / mrad

S1 θ_e spectrum



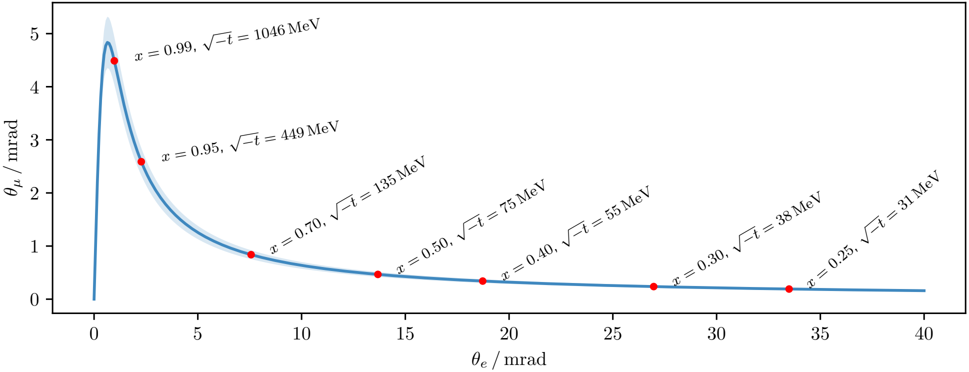


10.0
7.5
5.0
2.5
0.0
$d\sigma/d\theta$
LO
NNLO
0.004
0.002
0.000
$K-1$
signal
NLO
NNLO
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad

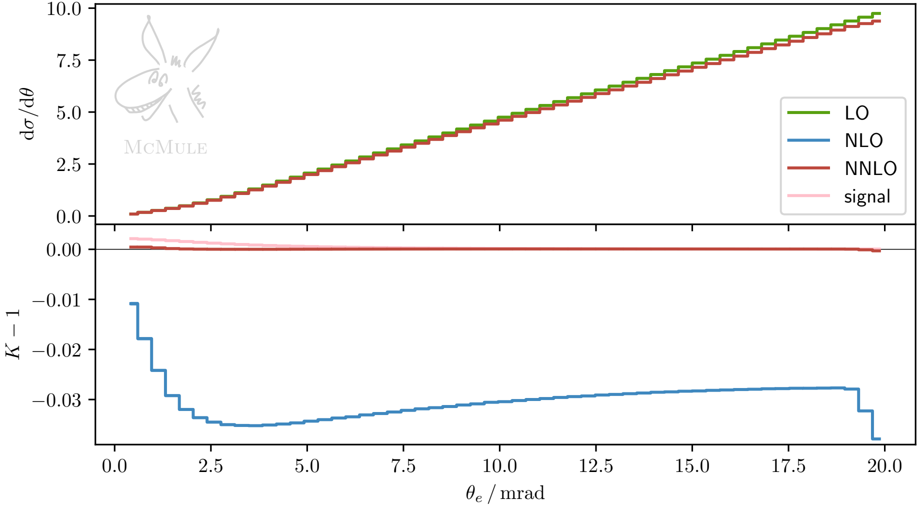

10.0
7.5
5.0
2.5
0.0
$d\sigma/d\theta$
McMule
LO
NNLO
0.004
0.002
0.000
$K - 1$
signal
NLO
NNLO
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad

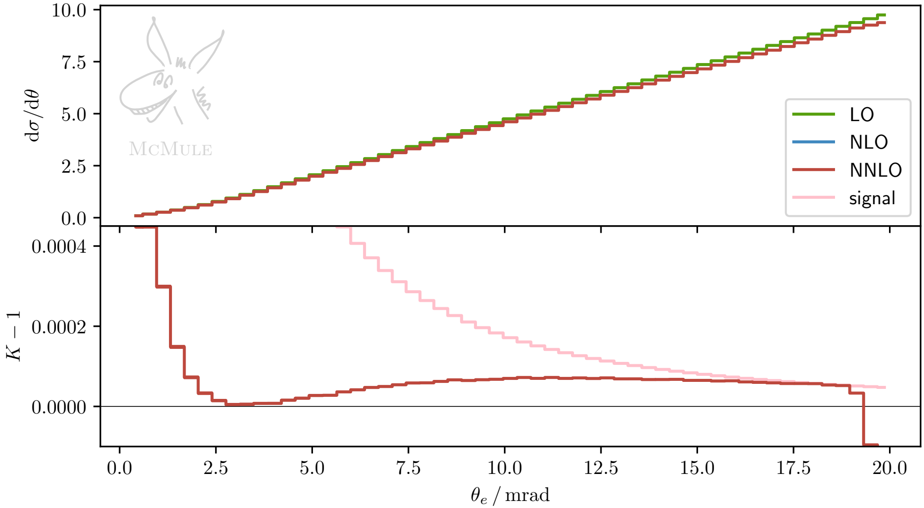
this clearly isn't working

- at this rate ($\sim 10\%$ NLO, $\sim 0.1\%$ NNLO), we would need N^{4}LO to reach 10^{-5}
- most of this is due to hard radition
- S2: same as S1 + needs to be in the band



S2 θ_e spectrum




10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
LO
NLO
NNLO
signal
0.0004
0.0002
0.0000
K − 1
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θe / mrad

VVV

- for $ee \to \gamma^*(\to \mu\mu)$: HQFF known [Fael, Lange, Schönwald, Steinhauser 22]
- for $ee \to \mu\mu$: massification (known) $\times$ massless (expected)

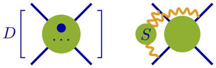
RVV

- full mass dependence unlikely (DiffExp-style is too slow for Monte Carlo)
- massless known from three-jet production
- massification...?

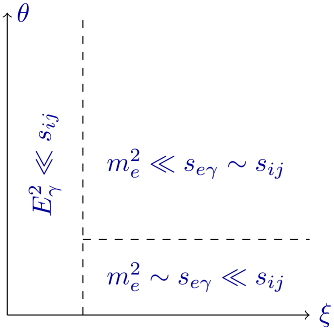
RRV

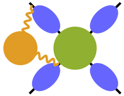
- OpenLoops + NTS

massifying real corrections

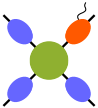


NTS expansion





massification



jettification

future of the mule

started and definitely happening

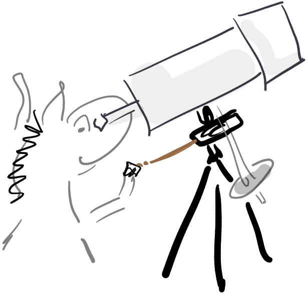
- approximate NNLO for $ee \to \mu\mu\gamma$ from $pp \to 2j + \gamma$ [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia 23]
- EW corrections processes in McMule in LEFT
- dispersive FsQED for $ee \to \pi\pi(\gamma)$ in McMule

pieces missing, could be fudged

- $ee \to \gamma^*$ at N^{3}LO (two-loop jettification missing)
- auto-generate matrix elements with NTS $\to$ beyond NNLO (better understanding of virtual helpful)

other plans (from concrete to vague)

- event generation with cellular resampling
- exclusive exponentiation (EEX), matched at NNLO
- EEX+NTS



f.l.t.r.: S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), V.Sharkovska (Zurich & PSI), S.Gündogdu (Zurich & PSI), D. Moreno (PSI), A.Coutinho (IFIC), Y.Ulrich (Liverpool), D. Radic (Zurich & PSI), L.Naterop (Zurich & PSI), M.Rocco (Turin)
not shown: F.Hagelstein (Mainz), N.Schalch (Oxford), T.Engel (Freiburg), A.Gurgone (Pavia), P.Banerjee (Cosenza)

McMule

`mule-tools.gitlab.io`