

TUM/MPP Collider Phenomenology Seminar

Towards QED at N³LO

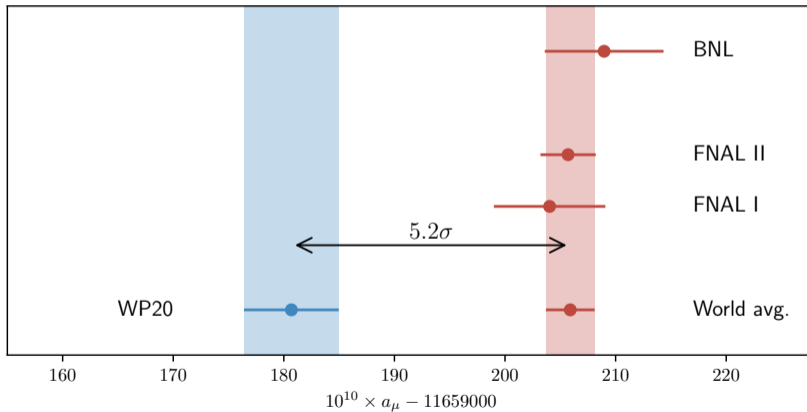
Yannick Ulrich

AEC, University of Bern

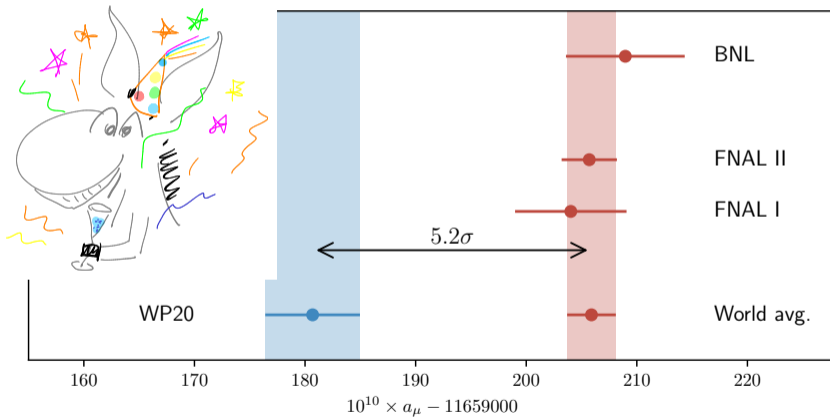
3 JULY 2024










- where & why do we need QED corrections?
- why do we need (partial) N³LO?
- what tools are needed for this?
- what to expect?
- some results from McMule

most precise measurement of $g - 2$

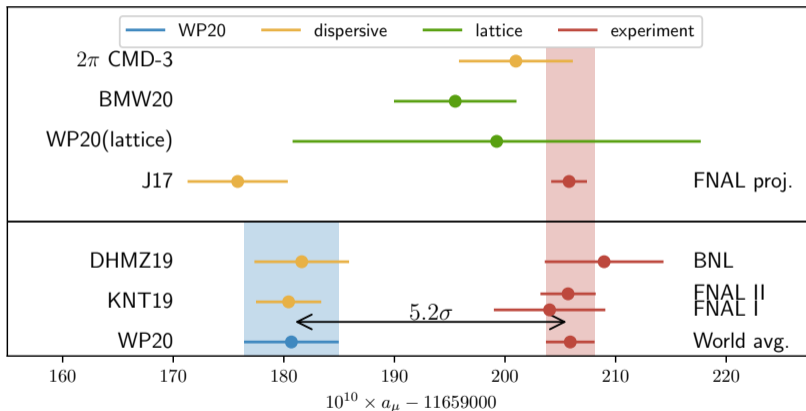


most precise measurement of $g - 2$



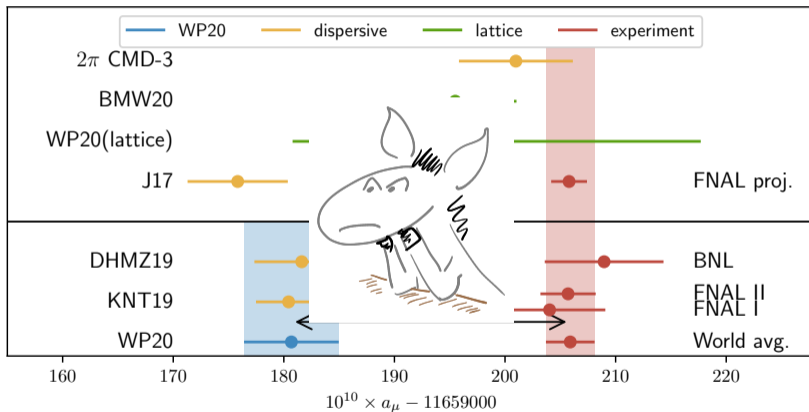
	value	diagrams
QED 1-loop	$\alpha/2\pi = 11\,614\,097.3$	
QED 2-loop	-17 723.1	 
QED 3-loop	148.0	 
more QED	-0.5	+ 3 others + 1 conspiracy theory + 70 others
EW	15.3	 
HVP	684.5(4.0)	+ others
HLbL	9.2(1.7)	 
total	11 659 181.0(4.3)	[g - 2 white paper 20]
FNAL+BNL	11 659 206.2(4.0)	

largest source of uncertainty & non-perturbative



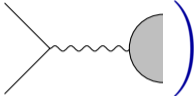
this problem is bigger than $g - 2$! [CMD-3 23] [BMW 20]

largest source of uncertainty & non-perturbative

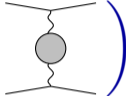


this problem is bigger than $g - 2$! [CMD-3 23] [BMW 20]

time-like in $ee \rightarrow \text{hadrons}$

$$\int ds \left(K(s) \text{  \right)$$

space-like in $e\mu \rightarrow e\mu$

$$\int dt \left(K'(t) \text{  \right)$$

time-like in $ee \rightarrow \text{hadrons}$

$$\int ds \left(K(s) \text{ [Diagram: } ee \rightarrow \text{hadrons via photon] } \right)$$

space-like in $e\mu \rightarrow e\mu$

$$\int dt \left(K'(t) \text{ [Diagram: } e\mu \rightarrow e\mu \text{ via photon] } \right)$$

... but what actually happens ...

radiative return measurement

$$\int ds \left(K(s) \text{ [Diagram: } ee \rightarrow \text{hadrons with radiative return] } \right)$$

loop-induced process

$$\int dt K'(t) \left(\text{ [Diagram: } e\mu \rightarrow e\mu \text{ with loop corrections] } \right)$$

radiative corrections are vital

time-like in $ee \rightarrow \text{hadrons}$

$$\int ds \left(K(s) \text{ [diagram: } ee \rightarrow \text{hadrons via photon] } \right)$$

space-like in $e\mu \rightarrow e\mu$

$$\int dt \left(K'(t) \text{ [diagram: } e\mu \rightarrow e\mu \text{ via photon] } \right)$$

... but what actually happens ...

radiative return measurement

$$\int ds \left(K(s) \frac{\text{[diagram: } ee \rightarrow \text{hadrons via photon with radiative return]}}{\text{[diagram: } ee \rightarrow \text{hadrons via photon]}} \right)$$

loop-induced process

$$\int dt K'(t) \left(\text{[diagram: } e\mu \rightarrow e\mu \text{ via photon with loop corrections] } \right)$$

radiative corrections are vital


benefiting from LHC technology where possible

- soft resummation: CEEX (\rightarrow improved YFS exponentiation)
- collinear resummation: parton shower & structure functions
- $2 \rightarrow 2$ with mass dependence at NNLO
 \Rightarrow precision: $\mathcal{O}(10^{-4})$

MUonE needs 10^{-5}

- $2 \rightarrow 3$ with mass dependence at NLO
 \Rightarrow precision: $\mathcal{O}(\text{few} \times 10^{-3})$

radiative return needs NNLO for kinematics

- pion final states: often only very simplified models $|F_\pi(s)|^2 \times$ 

full hadronic model needed

just like @ the LHC ...

amplitude



implementation



cross section

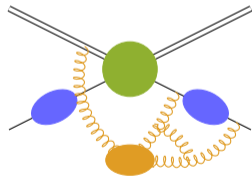
just like @ the LHC ...

... except fermion masses are physical \Rightarrow need massive amplitude

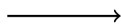
- they are also small \rightarrow we can drop terms $\sim \left(\frac{\alpha}{\pi}\right)^2 \log \frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$
- based on SCET factorisation & method of regions [Penin 06; Mitov, Moch 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g. $e\mu \rightarrow e\mu$ at two-loop:

$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$

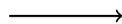
- **soft**: process-dependent $\mathcal{S} = 1 + \text{fermion loops}$
 \rightarrow compute separately to combine with hadron loops
- **collinear**: universal Z , converts $1/\epsilon \rightarrow \log(m^2/Q^2)$



amplitude



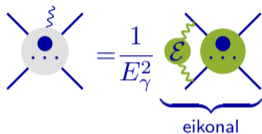
implementation



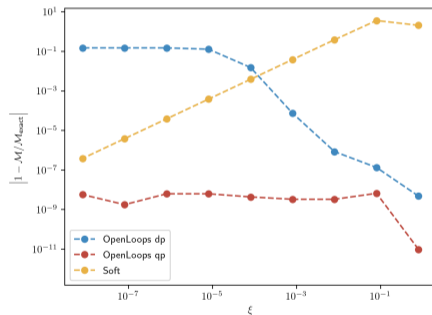
cross section

just like @ the LHC ...

... except the real-virtual can be delicate b/c it's more exclusive



$$+ \mathcal{O}(E_\gamma^{-1})$$



amplitude



implementation

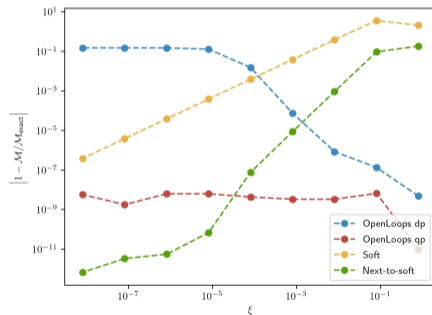


cross section

just like @ the LHC ...

... except the real-virtual can be delicate b/c it's more exclusive

$$\begin{aligned}
 & \text{Amplitude} = \frac{1}{E_\gamma^2} \underbrace{\text{eikonal}} + \frac{1}{E_\gamma} \left\{ \underbrace{D \left[\text{LBK} \right]} + \underbrace{\text{soft function}} \right\} + \mathcal{O}(E_\gamma^0)
 \end{aligned}$$



amplitude

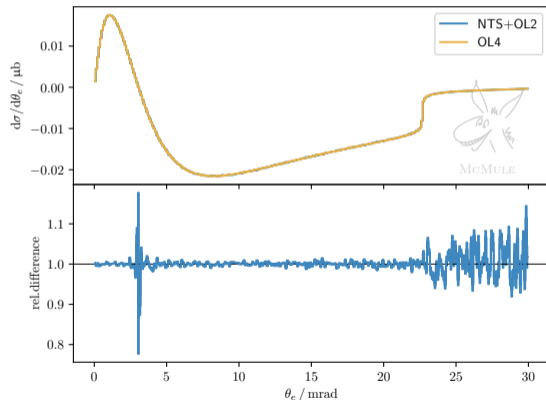


implementation



cross section

test next-to-soft stabilisation vs OL4 (OpenLoops quad) for $\mu e \rightarrow \mu e$ real-virtual



- same statistics, same result
 - 70 days vs 4 days
 - integrated results for different cuts
- ⇒ this is **not** an approximation but a numerical tool

NTS	OL4
-0.29268(4)	-0.29267(4)
-0.44789(6)	-0.44778(6)
-0.64662(9)	-0.64649(9)

just like @ the LHC ...

- universal soft limit $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E} \mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f} = \text{finite}$

use this to construct an all-order subtraction scheme FKS^ℓ [Engel, Signer, YU 19]

$$\underbrace{\int d\Phi_\gamma}_{\text{divergent and complicated}} \left[\text{diagram with grey circle} \right] = \underbrace{\int d\Phi_\gamma}_{\text{complicated but finite}} \left(\text{diagram with grey circle} - \text{diagram with green circle} \right) + \underbrace{\int d\Phi_\gamma}_{\text{divergent but easy}} \left[\text{diagram with green circle} \right]$$

amplitude



implementation



cross section

an effort to study & improve the state-of-the-art for $ee \rightarrow XX$

- next generation of [RadioMonteCarlow 0912.0749]
- calculate standard candles for $ee \rightarrow ee, \mu\mu, \pi\pi$ for various scenarios (scan & radiative return)
 - S0.7 (\sim CMD, $\sqrt{s} = 0.7$ GeV):
 $1 \leq \theta_{\text{avg}} \leq \pi - 1$, $|\vec{p}| > 0.45\sqrt{s}$,
 $|\phi^+ - \phi^-| - \pi| < 0.15$, $|\theta^+ - \theta^- - \pi| < 0.25$
 - LA1 (\sim KLOE, $\sqrt{s} = 1.02$ GeV):
 $50^\circ \leq \theta^\pm \leq 130^\circ$,
 $|p_z| > 90$ MeV \vee $|p_\perp| > 160$ MeV,
 $50^\circ \leq \theta_\gamma \leq 130^\circ$, $E_\gamma > 20$ MeV,
 $0.1 \text{ GeV}^2 \leq M_{\mu\mu}^2 \leq 0.85 \text{ GeV}^2$

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THE EUROPEAN
PHYSICAL JOURNAL C

Review

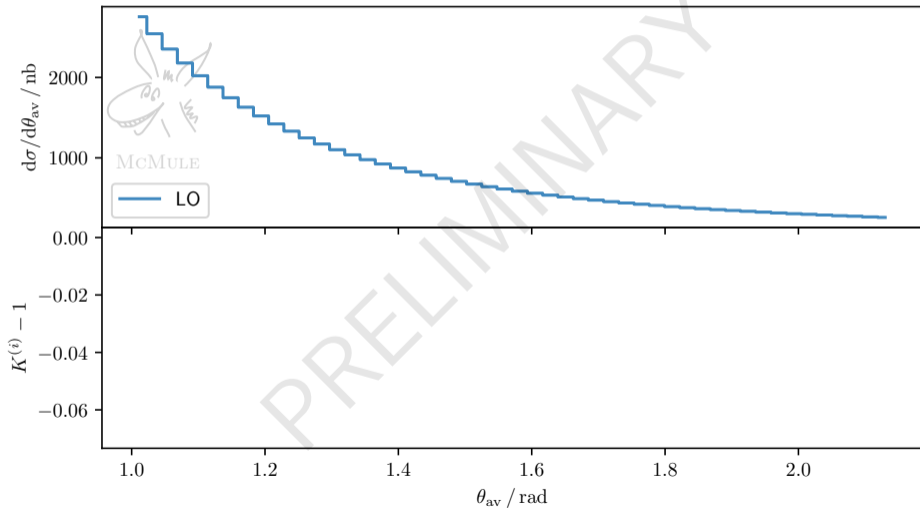
Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

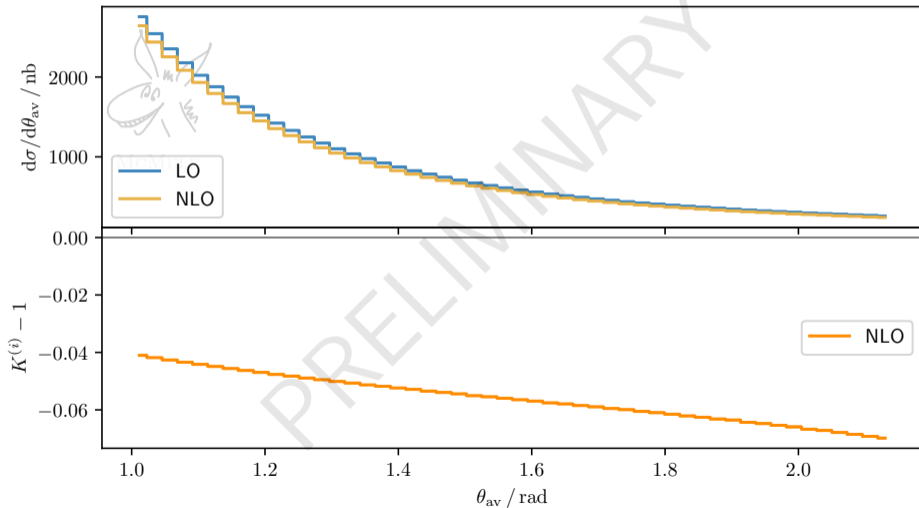
S. Actis²⁰, A. Arbuzov⁶⁹, G. Balossini^{22,23}, P. Beltrame²³, C. Bignamini^{23,23}, R. Bonciani²⁴, C.M. Carloni Calame²⁰, V. Cherepanov²⁵, M. Czakon⁶, H. Czyz^{26,61}, A. Denig²⁷, S. Eidelman^{28,64}, G.V. Felstovitch^{29,64}, A. Ferroglia³, J. Gluza⁶, A. Gracichinski³⁰, M. Gzella³¹, A. Haefliger³², F. Ignotov³³, S. Jadach³⁴, F. Jegerlehner^{35,61}, A. Kalinowski³⁶, W. Kluge³⁷, A. Korchin³⁸, J.H. Kühn³⁹, E.A. Kuraev⁴⁰, P. Laktka⁴¹, P. Mastrolia⁴², G. Montagna^{43,64}, S.E. Müller²², F. Nguyen⁴⁴, O. Nicrosini⁴⁵, D. Nomura^{46,47}, G. Pakhlova⁴⁸, G. Panzeri⁴¹, M. Passera⁴⁹, A. Penin⁵⁰, F. Piccinini⁵¹, W. Placzek⁵², T. Przewiński⁵³, E. Remiddi⁵⁴, T. Riemann⁵⁵, G. Rodrigo⁵⁶, P. Ruiz²⁷, O. Shakhmurova⁵⁷, C.P. Shen⁵⁸, A.L. Sibidanov⁵⁹, T. Tselmeir⁶⁰, L. Trentadue^{60,61}, G. Venanzoni^{11,62}, J.J. van der Bij⁶³, P. Wang⁶⁴, B.F.L. Ward⁶⁵, Z. Was⁶⁶, M. Worek^{67,68}, C.Z. Yuan⁶⁸

¹Institut für Theoretische Physik E, RWTH Aachen University, 52056 Aachen, Germany
²Institut für High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
³Institut für Physik Humboldt-Universität zu Berlin, 12489 Berlin, Germany
⁴Dipartimento di Fisica dell'Università di Bologna, 40126 Bologna, Italy
⁵INFN, Sezione di Bologna, 40126 Bologna, Italy
⁶The Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, Reymonta 4, 30-059 Cracow, Poland
⁷Maria Skłodowska Curie Institute of Physics, Jagiellonian University, Reymonta 4, 30-059 Cracow, Poland
⁸Institute of Nuclear Physics Polish Academy of Sciences, 31524 Cracow, Poland
⁹Bogdan-Labruna Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia
¹⁰Department of Physics, University of Alberta, Edmonton, AB T6G 2G1, Canada
¹¹Laboratori Nazionali di Frascati dell'INFN, 00044 Frascati, Italy
¹²Physikalisches Institut, Albert-Ludwigs-Universität Freiburg, 79104 Freiburg, Germany
¹³Physics Department, CERN, 1211 Geneva, Switzerland
¹⁴Theory Department, CERN, 1211 Geneva, Switzerland
¹⁵Laboratoire de Physique Subatomique et de Cosmologie, Université Joseph Fourier/CNRS-IN2P3/INPG, 38026 Grenoble, France
¹⁶University of Koval, Hradec Králové, 50002, Czechia
¹⁷Institut für Experimentelle Kernphysik, Universität Karlsruhe, 76021 Karlsruhe, Germany
¹⁸Institut für Theoretische Teilchenphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany
¹⁹Institute of Physics, University of Silesia, 40007 Katowice, Poland
²⁰National Science Center "Kharkov Institute of Physics and Technology", 61108 Kharkov, Ukraine
²¹Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK
²²Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, 55128 Mainz, Germany
²³Institut für Physik (THEP), Johannes Gutenberg-Universität, 55099 Mainz, Germany
²⁴Institute for Theoretical and Experimental Physics, Moscow, Russia
²⁵Russian Institute of Nuclear Physics, 630090 Novosibirsk, Russia
²⁶Novosibirsk State University, 630090 Novosibirsk, Russia
²⁷Laboratoire de Physique Théorique (UMR 8627), Université de Paris-Sud XI, Bâtiment 210, 91405 Orsay Cedex, France
²⁸INFN, Sezione di Padova, 35131 Padova, Italy
²⁹L.R. Ecole Polytechnique, 91128 Palaiseau, France
³⁰Dipartimento di Fisica, Università di Parma, 43100 Parma, Italy
³¹INFN, Gruppo Collegato di Parma, 43100 Parma, Italy
³²Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, 27100 Pavia, Italy
³³INFN, Sezione di Pavia, 27100 Pavia, Italy
³⁴Dipartimento di Fisica dell'Università "Roma Tre" and INFN, Sezione di Roma Tre, 00146 Rome, Italy
³⁵School of Physics and Astronomy, University of Southampton, Southampton SO9 1NH, UK
³⁶Theory Center KEK, Tsukuba, Ibaraki 305-0801, Japan
³⁷Instituto de Física Corpuscular (IFC), Centro mixto UVEG-CSIC, Edificio Instrumentos de Investigación, Apartado de Correos 22085, 46071 Valencia, Spain
³⁸Paul Scherrer Institut, Würenlingen and Villigen, 5232 Villigen PSI, Switzerland
³⁹Department of Physics, Baylor University, Waco, TX 76798-7316, USA
⁴⁰Physikalisches C, Heinrich-Heine-Universität Wuppertal, 42077 Wuppertal, Germany
⁴¹Deutsches Elektronen-Synchrotron, DESY, 15738 Zeuthen, Germany

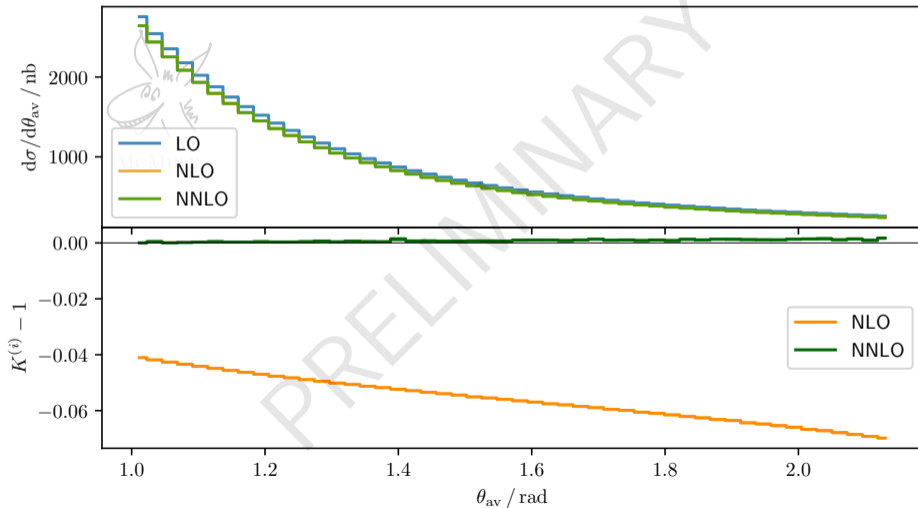
$e^-e^+ \rightarrow e^-e^+ @ S0.7$



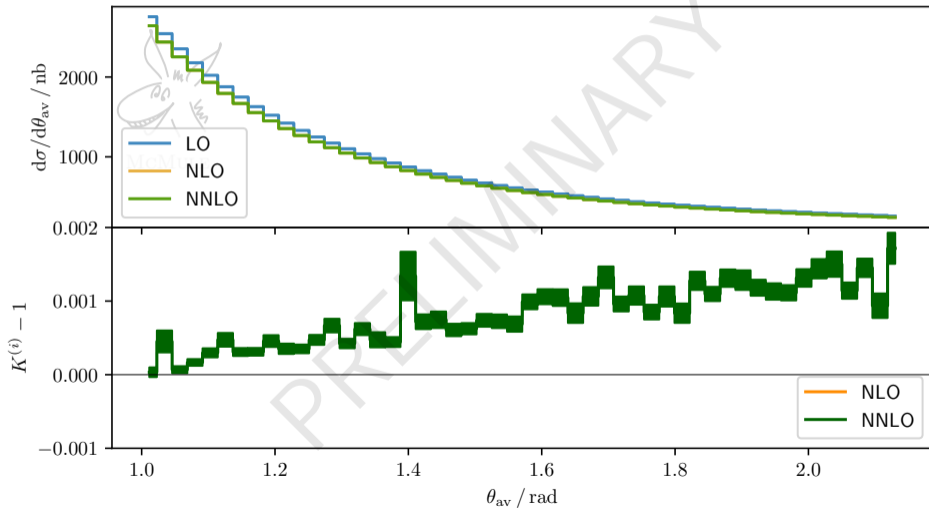
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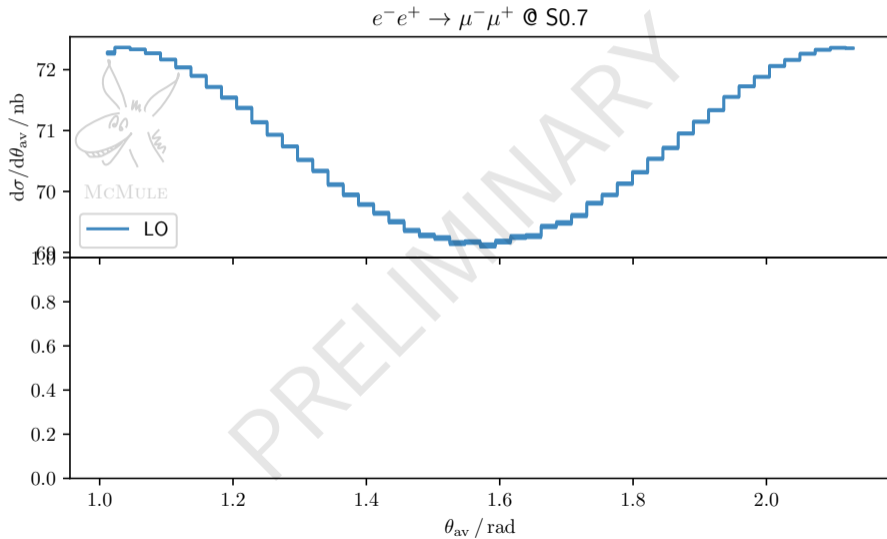


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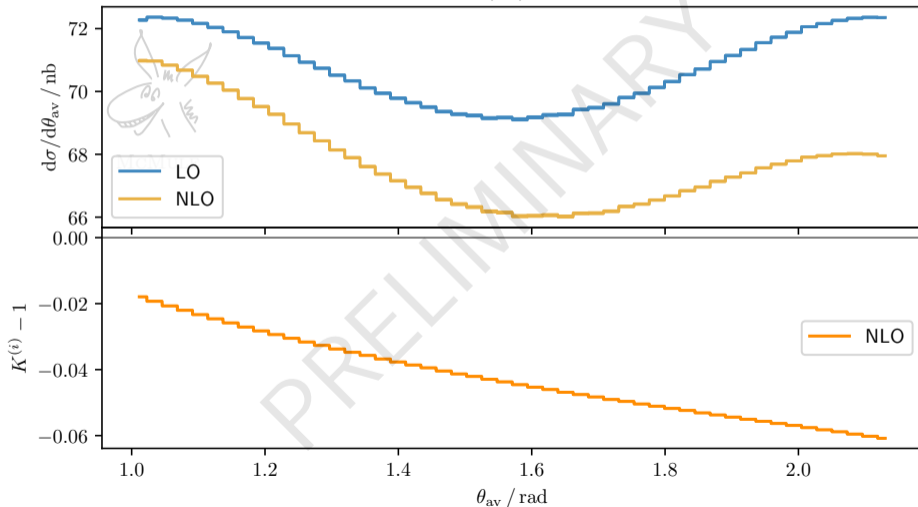


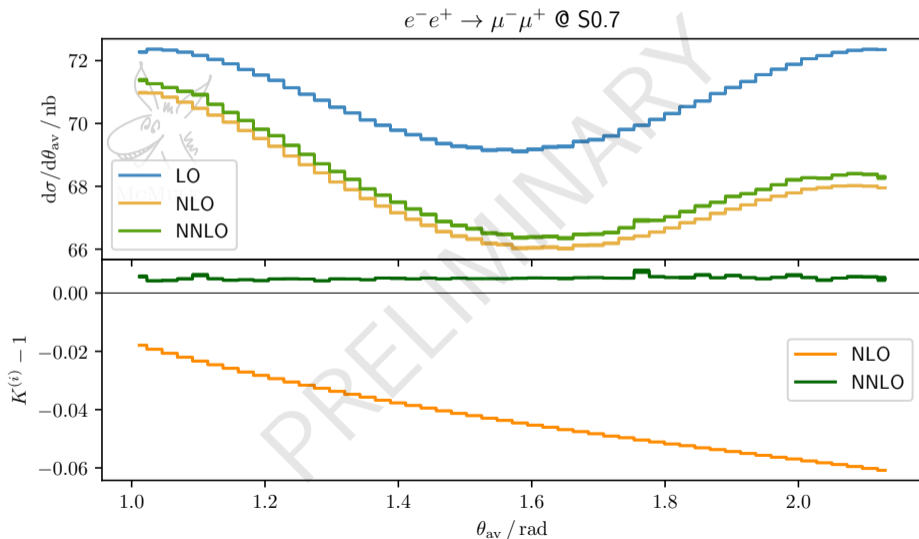
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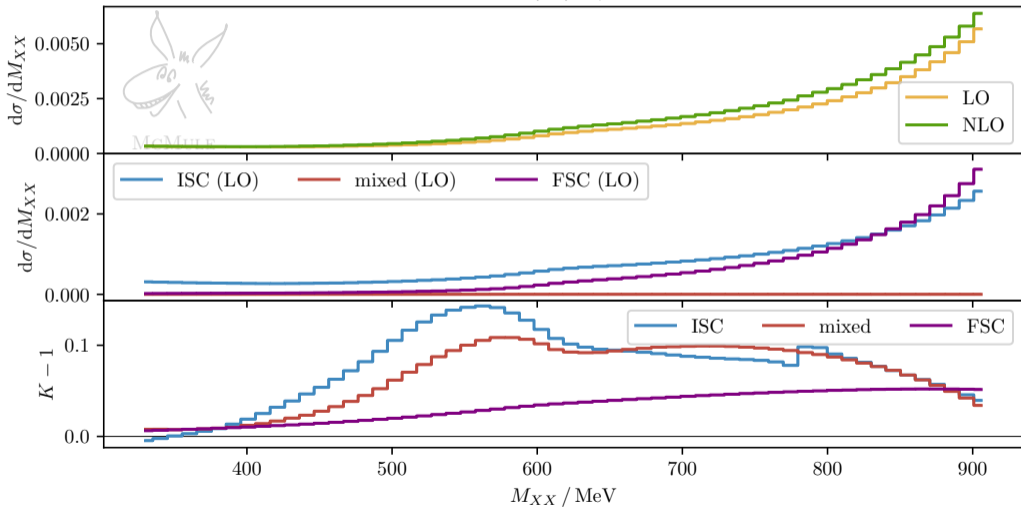


$e^-e^+ \rightarrow \mu^-\mu^+ @ S0.7$





$e^-e^+ \rightarrow \mu^-\mu^+\gamma$ @ LA1



- this pipeline also works for $2 \rightarrow 3$
- ... as long as $m_f^2 \ll s_{ij}$ (massification)

for $ee \rightarrow \mu\mu\gamma$

- photon is detected, i.e. hard & large angle
 - the simplest part ($ee \rightarrow \gamma\gamma^*(\rightarrow \mu\mu)$) with $m_e^2 \ll s_{ij}$
 - the full with $m_e^2 \sim m_\mu^2 \ll s_{ij}$ with amplitudes from $pp \rightarrow 2j + \gamma$ (WIP)
- \Rightarrow no theoretical showstoppers, fairly doable

VVV

- for $ee \rightarrow \gamma^*(\rightarrow \mu\mu)$: HQFF known [Fael, Lange, Schönwald, Steinhauser 22]
- for $ee \rightarrow \mu\mu$: massification (known) \times massless (expected)

RVV

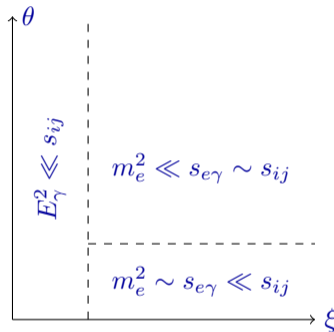
- full mass dependence unlikely (DiffExp-style is too slow for Monte Carlo)
- massless known from three-jet production
- massification...?

RRV

- OpenLoops + NTS



NTS expansion



massification



jettification


expand for $m_e^2 \sim p_e \cdot p_\gamma \ll p_e \cdot q \sim p_\gamma \cdot q$

- calculation in SCET
- two non-trivial scales: $(p_e \cdot p_\gamma)/m_e^2$ and $(p_e \cdot q)/(p_\gamma \cdot q)$
- integrals not regularised in DIMREG

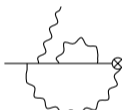
$$\frac{1}{\ell \cdot \bar{n}} \rightarrow \frac{1}{(\ell \cdot \bar{n})^{1+\eta}}$$

or

$$\frac{1}{\ell \cdot \bar{n}} \rightarrow \frac{1}{\ell \cdot \bar{n} + \Delta}$$

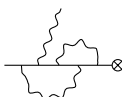


$$\sim \int \frac{1}{\ell_1 \cdot \bar{n}} \frac{1}{\ell_1^2} \frac{1}{\ell_2 \cdot \bar{n}} \frac{1}{\ell_2^2} \dots$$



$$\sim \int \frac{1}{\ell_1 \cdot \bar{n}} \frac{1}{\ell_1^2} \frac{1}{\ell_2^2} \dots$$

- either complicates the integrals
- final result J finite in η or Δ



$$\sim \int \frac{1}{\ell_1^2} \frac{1}{\ell_2^2} \dots$$

[WIP, Schalch, Engel, YU]

- the NNLO $2 \rightarrow 2$ era has arrived, also for QED
 - NNLO $2 \rightarrow 3$ possible for many things by adapting LHC results
 - (partial) N³LO possible in the near future
 - resumatation is vital to reach the target precision
- ⇒ plans for YFS-based EEX in McMule, PS-based in other codes (Babayaga/MESMER)



f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI),
S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg),
V.Sharkovska (Zurich & PSI), Y.Ulrich (Bern), A.Gurgone (Pavia)
not pictured: P.Banerjee (IIT Guwahati), D.Moreno (PSI), D.Radic (PSI)



McMule

mule-tools.gitlab.io