ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS
McMule

KIT Particle Physics Seminar

Towards QED at N^3LO

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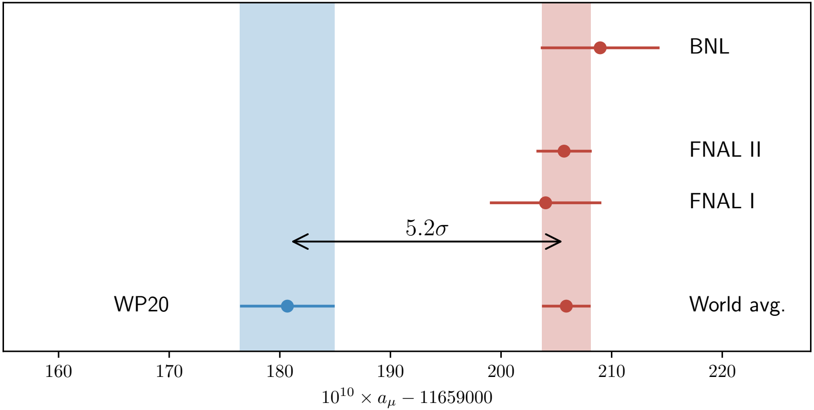
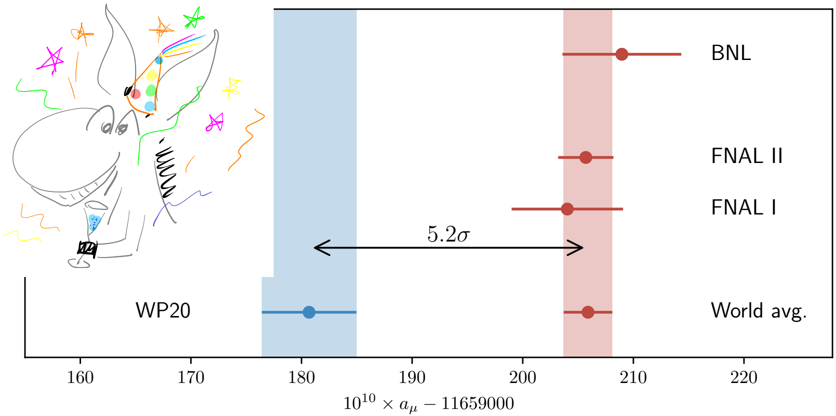
25 June 2024

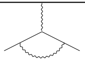
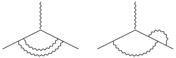
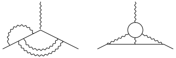
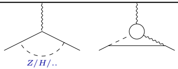
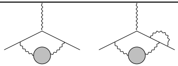
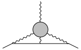
- where & why do we need QED corrections?
- why do we need (partial) $\mathrm{N}^3\mathrm{LO}$?
- what tools are needed for this?
- what to expect?
- some results from McMule

most precise measurement of $g-2$

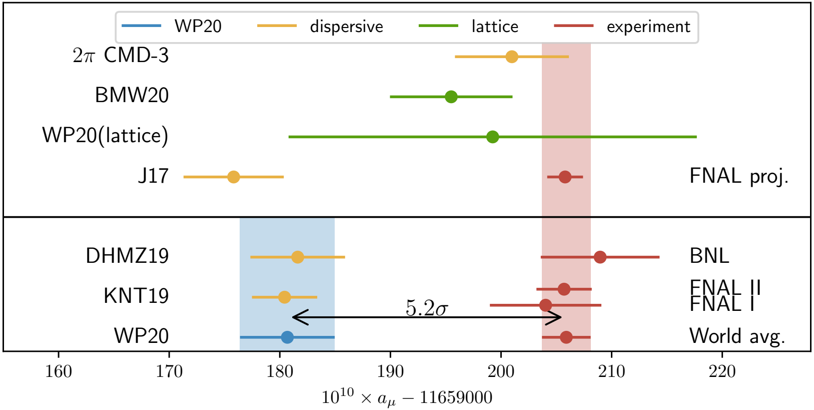


most precise measurement of $g-2$

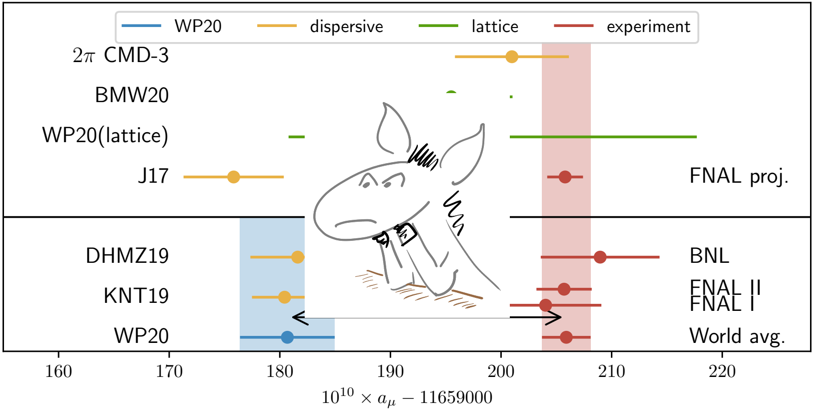


	value	diagrams
QED 1-loop	$\alpha/2\pi = 11\,614\,097.3$	
QED 2-loop	$-17\,723.1$	 + 3 others + 1 conspiracy theory
QED 3-loop	148.0	 + 70 others
more QED	-0.5	
EW	15.3	
HVP	$684.5(4.0)$	 + others
HLbL	$9.2(1.7)$	
total	$11\,659\,181.0(4.3)$	[$g-2$ white paper 20]
FNAL+BNL	$11\,659\,206.2(4.0)$	

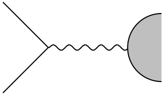
largest source of uncertainty & non-perturbative



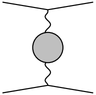
this problem is bigger than $g-2$! [CMD-3 23] [BMW 20] largest source of uncertainty & non-perturbative



this problem is bigger than $g-2$! [CMD-3 23] [BMW 20] time-like in $ee \to$ hadrons

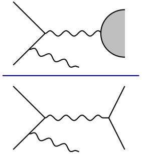
$$\int \mathrm{d}s \Big(K(s)$$  $$\Big)$$

space-like in $e\mu \to e\mu$

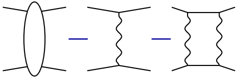
$$\int \mathrm{d}t \Big(K'(t)$$  $$\Big)$$

... but what actually happens ...

radiative return measurement

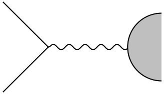
$$\int \mathrm{d}s \Big(K(s)$$  $$\Big)$$

loop-induced process

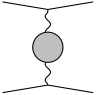
$$\int \mathrm{d}t \, K'(t) \Big($$  $$\Big)$$

radiative corrections are vital

time-like in $ee \to$ hadrons

$$\int \mathrm{d}s \Big(K(s)$$  $$\Big)$$

space-like in $e\mu \to e\mu$

$$\int \mathrm{d}t \Big(K'(t)$$  $$\Big)$$

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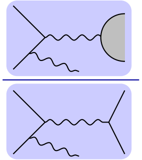
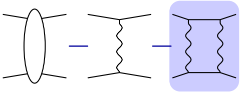
radiative return measurement

$$\int \mathrm{d}s \Big(K(s)$$  $$\Big)$$

loop-induced process

$$\int \mathrm{d}t\, K'(t) \Big($$  $$\Big)$$

radiative corrections are vital

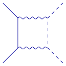
benefiting from LHC technology where possible

- soft resummation: CEEX ($\to$ improved YFS exponentiation)
- collinear resummation: parton shower & structure functions
- $2 \to 2$ with mass dependence at NNLO

  $\Rightarrow$ precision: $\mathcal{O}(10^{-4})$

  MUonE needs 10^{-5}
- $2 \to 3$ with mass dependence at NLO

  $\Rightarrow$ precision: $\mathcal{O}(\text{few} \times 10^{-3})$

  radiative return needs NNLO for kinematics
- pion final states: often only very simplified models $\left|F_\pi(s)\right|^2 \times$ 

  full hadronic model needed

just like @ the LHC ...

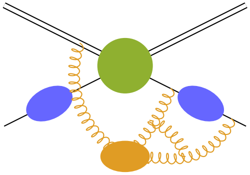
amplitude ⟶ implementation ⟶ cross section

just like @ the LHC ...

... except fermion masses are physical $\Rightarrow$ need massive amplitude

- they are also small $\rightarrow$ we can drop terms $\sim \left(\frac{\alpha}{\pi}\right)^2 \log\frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$
- based on SCET factorisation & method of regions [Penin 06; Mitov, Moch 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g. $e\mu \to e\mu$ at two-loop:

$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$



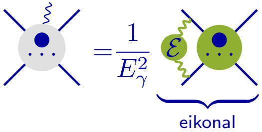
- soft: process-dependent $S = 1 +$ fermion loops
  $\rightarrow$ compute separately to combine with hadron loops
- collinear: universal Z, converts $1/\epsilon \to \log(m^2/Q^2)$

amplitude $\longrightarrow$ implementation $\longrightarrow$ cross section

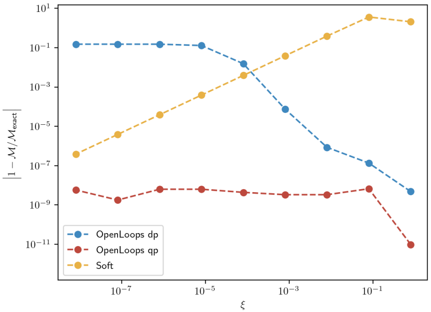
how does an NNLO QED calculation work?

just like @ the LHC ...

... except the real-virtual can be delicate b/c it's more exclusive



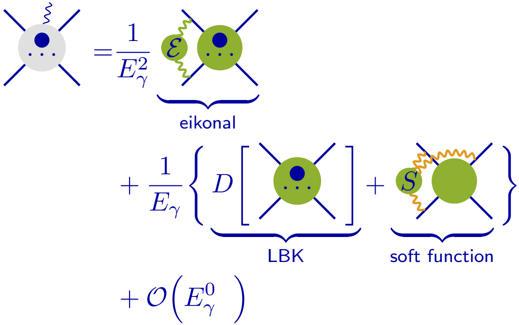
$$+\,\mathcal{O}\left(E_\gamma^{-1}\right)$$

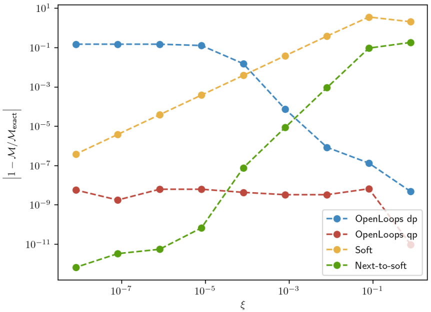


amplitude ⟶ implementation ⟶ cross section

just like @ the LHC ...

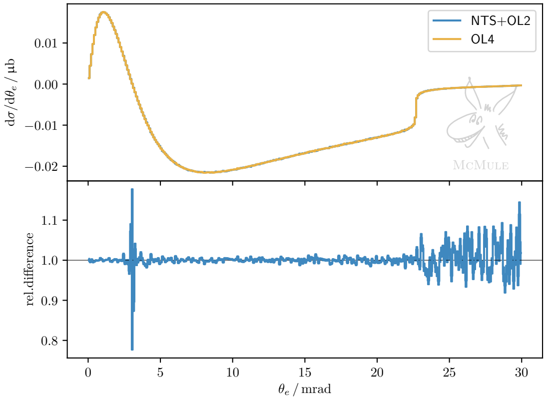
... except the real-virtual can be delicate b/c it's more exclusive





amplitude ⟶ implementation ⟶ cross section

test next-to-soft stabilisation vs OL4 (OpenLoops quad) for $\mu e \to \mu e$ real-virtual



- same statistics, same result
- 70 days vs 4 days
- integrated results for different cuts

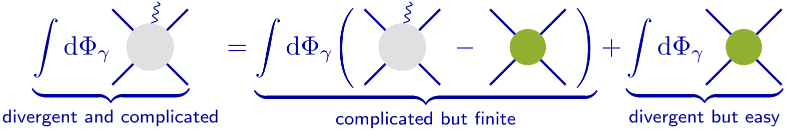
⇒ this is not an approximation but a numerical tool

NTS	OL4
$-0.29268(4)$	$-0.29267(4)$
$-0.44789(6)$	$-0.44778(6)$
$-0.64662(9)$	$-0.64649(9)$

just like @ the LHC ...

- universal soft limit $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f} =$ finite

use this to construct an all-order subtraction scheme FKS$^\ell$ [Engel, Signer, YU 19]



amplitude ⟶ implementation ⟶ cross section

an effort to study & improve the state-of-the-art for $ee \to XX$

- next generation of [RadioMonteCarlow 0912.0749]
- calculate standard candles for $ee \to ee, \mu\mu, \pi\pi$ for various scenarios (scan & radiative return)
  - S0.7 ($\sim$ CMD, $\sqrt{s} = 0.7\,\mathrm{GeV}$):
    $1 \le \theta_{\text{avg}} \le \pi - 1$, $|\vec{p}| > 0.45\sqrt{s}$,
    $||\phi^+ - \phi^-| - \pi| < 0.15$, $|\theta^+ - \theta^- - \pi| < 0.25$
  - LA1 ($\sim$ KLOE, $\sqrt{s} = 1.02\,\mathrm{GeV}$):
    $50° \le \theta^\pm \le 130°$,
    $|p_z| > 90\,\mathrm{MeV} \vee |p_\perp| > 160\,\mathrm{MeV}$,
    $50° \le \theta_\gamma \le 130°$, $E_\gamma > 20\,\mathrm{MeV}$,
    $0.1\,\mathrm{GeV}^2 \le M^2_{\mu\mu} \le 0.85\,\mathrm{GeV}^2$

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THE EUROPEAN PHYSICAL JOURNAL C

Review

Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

S. Actis[38], A. Arbuzov[9,e], G. Balossini[32,33], P. Beltrame[13], C. Bignamini[32,33], R. Bonciani[15], C.M. Carloni Calame[35], V. Cherepanov[25,26], M. Czakon[1], H. Czyż[19,a,f,i], A. Denig[22], S. Eidelman[25,26,g], G.V. Fedotovich[25,26,e], A. Ferroglia[23], J. Gluza[19], A. Grzelińska[8], M. Gunia[19], A. Hafner[22], F. Ignatov[25], S. Jadach[8], F. Jegerlehner[3,19,41], A. Kalinowski[29], W. Kluge[17], A. Korchin[20], J.H. Kühn[18], E.A. Kuraev[9], P. Lukin[25], P. Mastrolia[14], G. Montagna[32,33,b,d], S.E. Müller[22,f], F. Nguyen[34,d], O. Nicrosini[33], D. Nomura[36,h], G. Pakhlova[24], G. Pancheri[11], M. Passera[28], A. Penin[10], F. Piccinini[33], W. Płaczek[7], T. Przedzinski[6], E. Remiddi[4,5], T. Riemann[41], G. Rodrigo[37], P. Roig[27], O. Shekhovtsova[11], C.P. Shen[16], A.L. Sibidanov[25], T. Teubner[21,h], L. Trentadue[30,31], G. Venanzoni[11,c,i], J.J. van der Bij[12], P. Wang[2], B.F.L. Ward[39], Z. Was[8,g], M. Worek[40,19], C.Z. Yuan[2]

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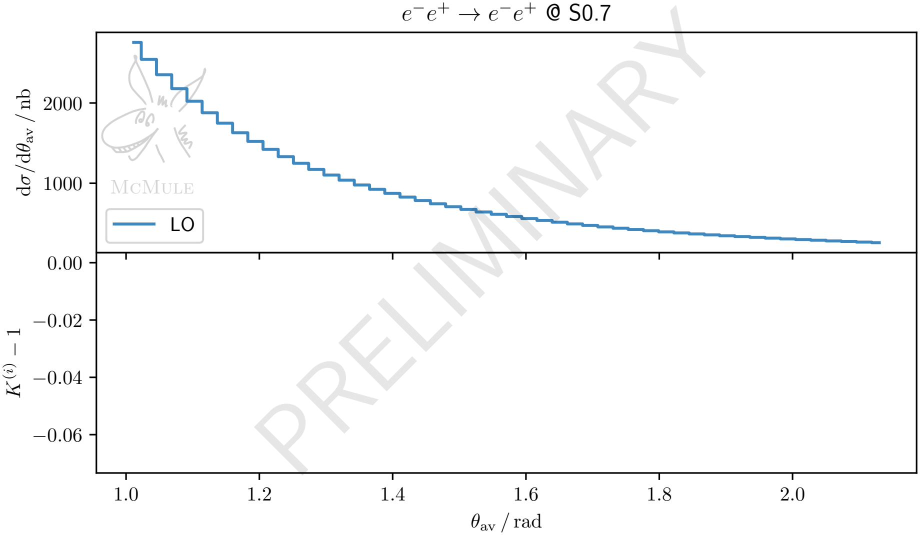
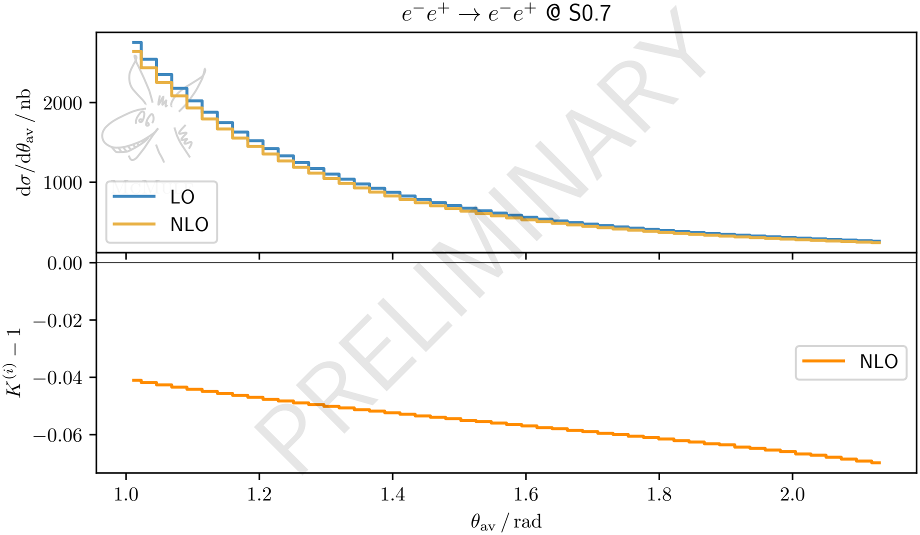
Springer



$e^-e^+ \to e^-e^+$ @ S0.7
$d\sigma/d\theta_{av}$ / nb
2000
1000
LO
McMule
PRELIMINARY
0.00
−0.02
−0.04
−0.06
$K^{(i)} - 1$
1.0
1.2
1.4
1.6
1.8
2.0
θ_{av} / rad



$e^-e^+ \to e^-e^+$ @ S0.7
PRELIMINARY
$d\sigma/d\theta_{av}$ / nb
2000
1000
LO
NLO
0.00
−0.02
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$K^{(i)} - 1$
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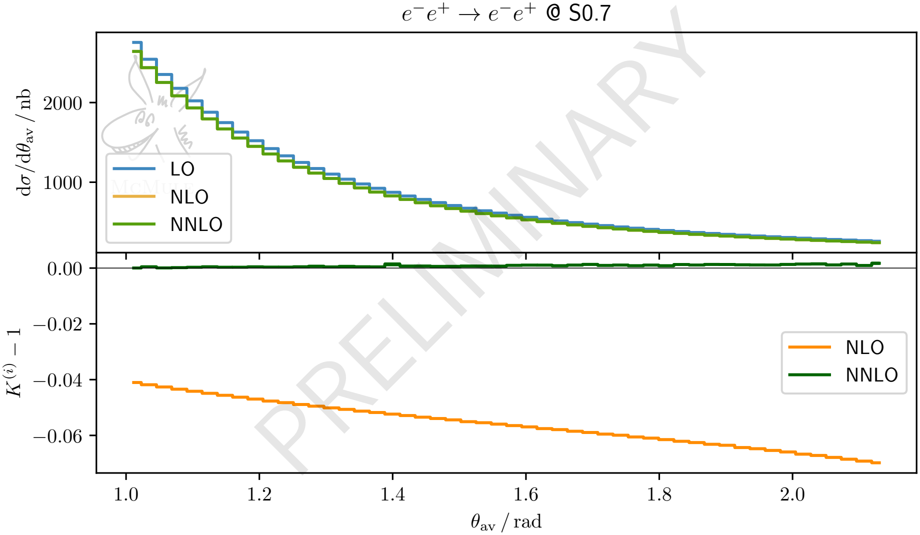

$e^-e^+ \to e^-e^+$ @ S0.7
$d\sigma/d\theta_{av}$ / nb
2000
1000
LO
NLO
NNLO
PRELIMINARY
0.00
−0.02
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$K^{(i)} - 1$
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1.0
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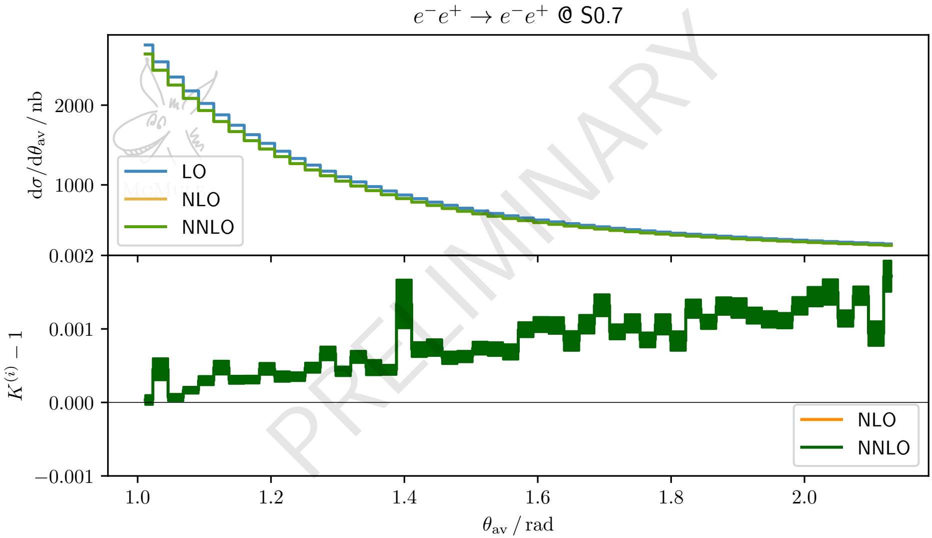
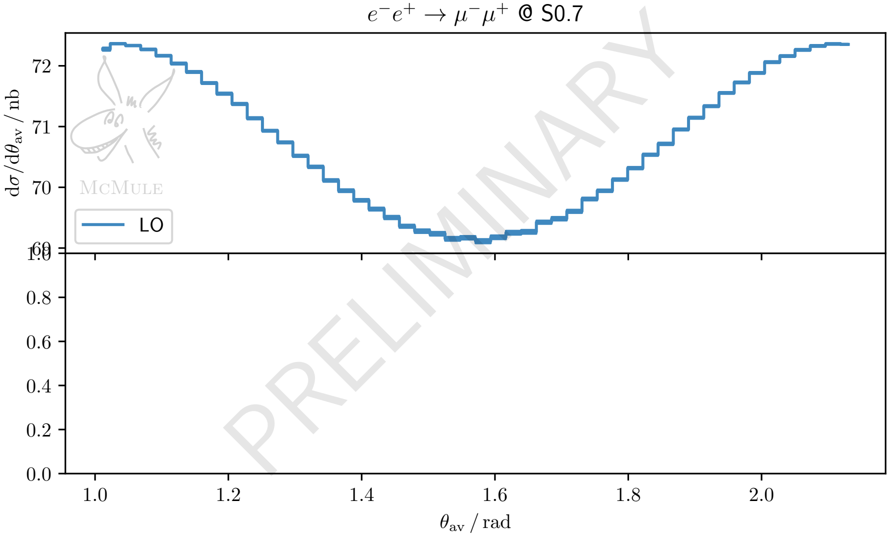
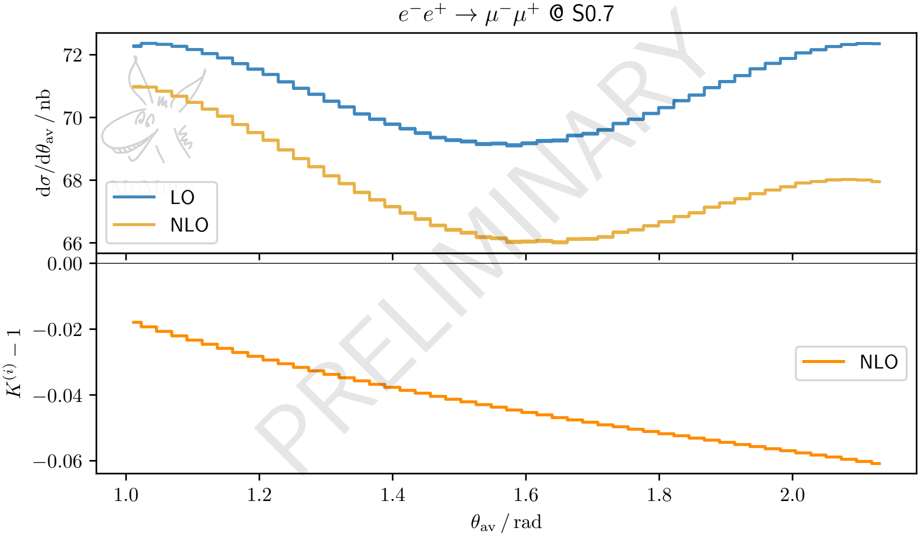
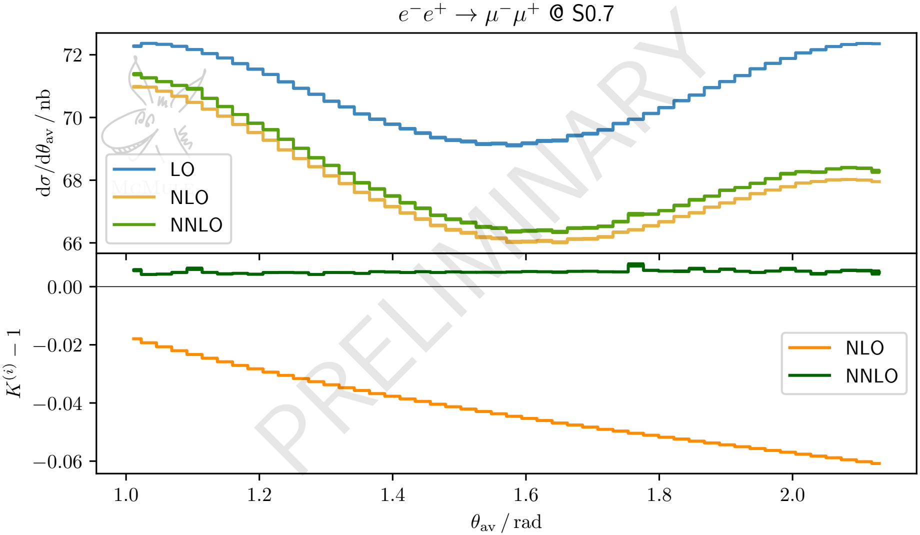
θ_{av} / rad


$e^-e^+ \to e^-e^+$ @ S0.7
$d\sigma/d\theta_{av}$ / nb
2000
1000
LO
NLO
NNLO
0.002
0.001
0.000
−0.001
$K^{(i)} - 1$
NLO
NNLO
1.0
1.2
1.4
1.6
1.8
2.0
θ_{av} / rad
PRELIMINARY

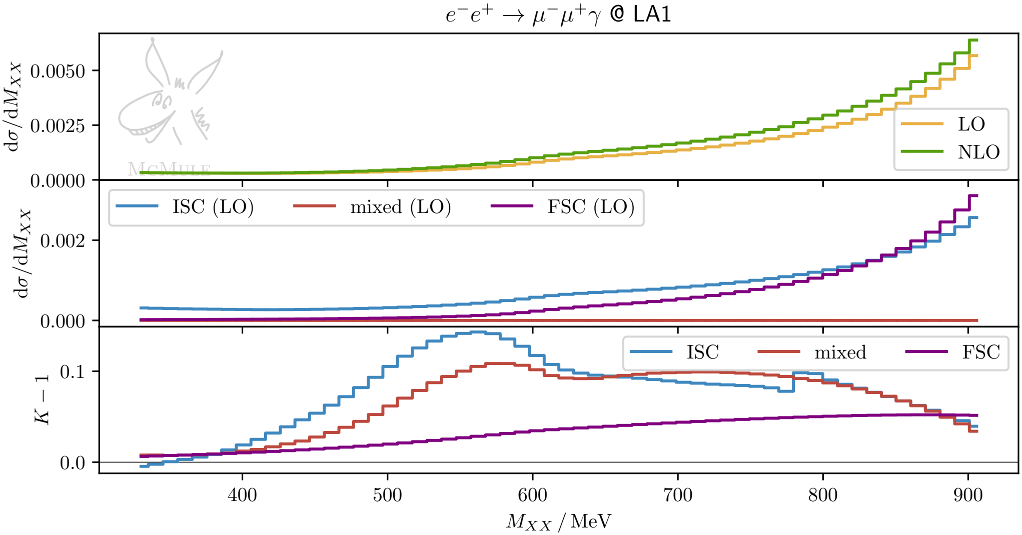

$e^-e^+ \to \mu^-\mu^+$ @ S0.7
$d\sigma/d\theta_{av}$ / nb
72
71
70
69
1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.2
1.4
1.6
1.8
2.0
θ_{av} / rad
LO
McMule
PRELIMINARY



$e^-e^+ \to \mu^-\mu^+$ @ S0.7
$d\sigma/d\theta_{av}$ / nb
LO
NLO
$K^{(i)} - 1$
NLO
θ_{av} / rad
PRELIMINARY
72
70
68
66
0.00
−0.02
−0.04
−0.06
1.0
1.2
1.4
1.6
1.8
2.0



$e^-e^+ \to \mu^-\mu^+$ @ S0.7
$d\sigma/d\theta_{av}$ / nb
72
70
68
66
LO
NLO
NNLO
$K^{(i)} - 1$
0.00
−0.02
−0.04
−0.06
NLO
NNLO
1.0
1.2
1.4
1.6
1.8
2.0
θ_{av} / rad
PRELIMINARY


$e^-e^+ \to \mu^-\mu^+\gamma$ @ LA1
$d\sigma/dM_{XX}$
0.0050
0.0025
0.0000
LO
NLO
ISC (LO)
mixed (LO)
FSC (LO)
0.002
0.000
$K-1$
0.1
0.0
ISC
mixed
FSC
400
500
600
700
800
900
M_{XX} / MeV

NNLO for $2 \to 3$

- this pipeline also works for $2 \to 3$
- ... as longs as $m_f^2 \ll s_{ij}$ (massification)

for $ee \to \mu\mu\gamma$

- photon is detected, i.e. hard & large angle
- the simplest part ($ee \to \gamma\gamma^*(\to \mu\mu)$) with $m_e^2 \ll s_{ij}$
- the full with $m_e^2 \sim m_\mu^2 \ll s_{ij}$ with amplitudes from $pp \to 2j + \gamma$ (WIP)

$\Rightarrow$ no theoretical showstoppers, fairly doable

VVV

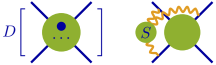
- for $ee \to \gamma^*(\to \mu\mu)$: HQFF known [Fael, Lange, Schönwald, Steinhauser 22]
- for $ee \to \mu\mu$: massification (known) $\times$ massless (expected)

RVV

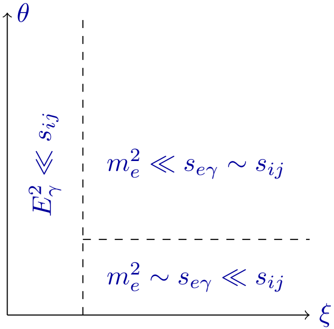
- full mass dependence unlikely (DiffExp-style is too slow for Monte Carlo)
- massless known from three-jet production
- massification...?

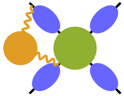
RRV

- OpenLoops + NTS

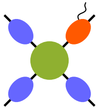


NTS expansion





massification



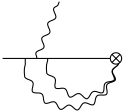
jettification

expand for $m_e^2 \sim p_e \cdot p_\gamma \ll p_e \cdot q \sim p_\gamma \cdot q$

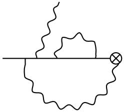
- calculation in SCET
- two non-trivial scales: $(p_e \cdot p_\gamma)/m_e^2$ and $(p_e \cdot q)/(p_\gamma \cdot q)$
- integrals not regularised in DIMREG

$$\frac{1}{\ell \cdot \bar{n}} \to \frac{1}{(\ell \cdot \bar{n})^{1+\eta}} \qquad \text{or} \qquad \frac{1}{\ell \cdot \bar{n}} \to \frac{1}{\ell \cdot \bar{n} + \Delta}$$

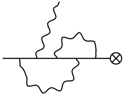
- either complicates the integrals
- final result J finite in η or Δ



$$\sim \int \frac{1}{\ell_1 \cdot \bar{n}} \frac{1}{\ell_1^2} \frac{1}{\ell_2 \cdot \bar{n}} \frac{1}{\ell_2^2} \cdots$$



$$\sim \int \frac{1}{\ell_1 \cdot \bar{n}} \frac{1}{\ell_1^2} \frac{1}{\ell_2^2} \cdots$$



$$\sim \int \frac{1}{\ell_1^2} \frac{1}{\ell_2^2} \cdots$$

[WIP, Schalch, Engel, YU]

- the NNLO $2 \to 2$ era has arrived, also for QED
- NNLO $2 \to 3$ possible for many things by adapting LHC results
- (partial) N^{3}LO possible in the near future
- needs to be matched to resummation

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McMule

`mule-tools.gitlab.io`