

QCD Seminar

μ - e scattering at 10ppm

Yannick Ulrich

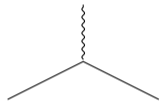
AEC, University of Bern

27 MAY 2024

- magnetic moment of a charged lepton: $\vec{\mu} = g \frac{e}{2m} \vec{S}$

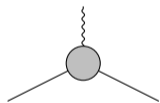
- Dirac: $g_{\mu}^{\text{Dirac}} = 2$

$$(-ie)\bar{u}\gamma^{\mu}u =$$

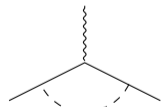


- SM quantum corrections: $g_{\mu}^{\text{SM}} = 2 \times (1 + a_{\mu}) = 2 \times (1 + F_2(0))$

$$(-ie)\bar{u}\left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2)\frac{i\sigma^{\mu\nu}Q_{\nu}}{2m}\right]u =$$

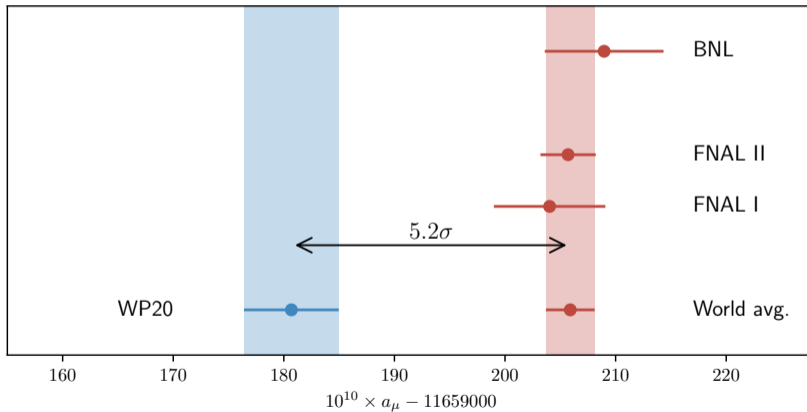


- BSM quantum corrections: $g_{\mu}^{\text{BSM}} \sim g_{\mu}^{\text{exp}} - g_{\mu}^{\text{SM}}$

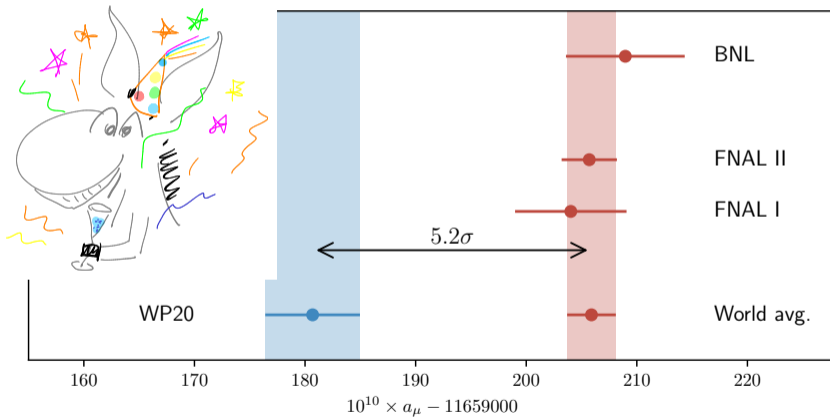












(insert favourite BSM)

most precise measurement of $g - 2$

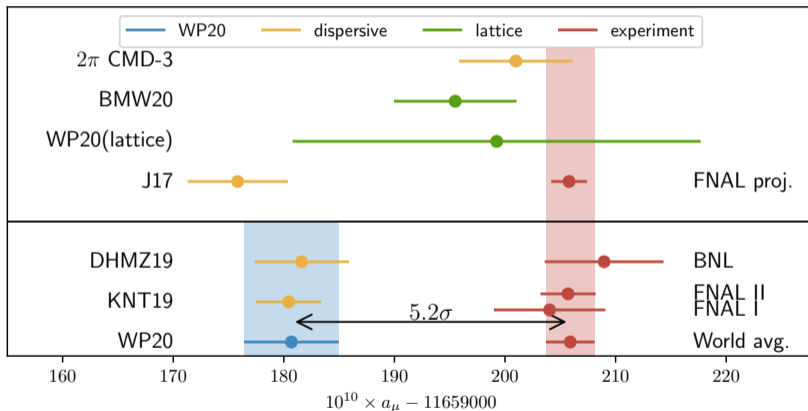


most precise measurement of $g - 2$



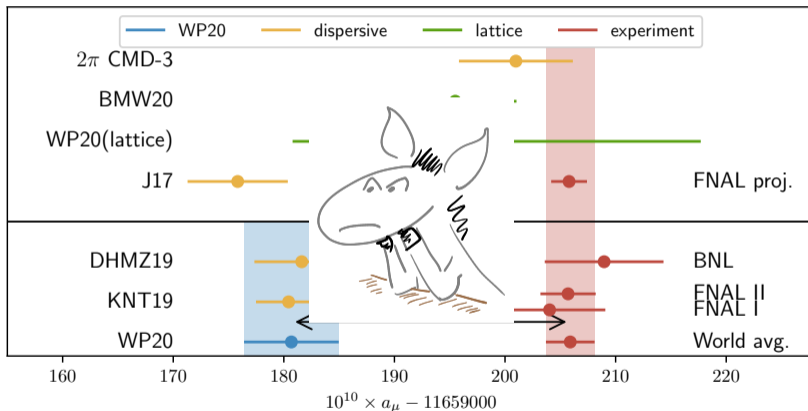
	value	diagrams
QED 1-loop	$\alpha/2\pi = 116\,140\,973$	
QED 2-loop	-177 231	 
QED 3-loop	1 480	 
more QED	-5	+ 3 others + 1 conspiracy theory + 70 others
EW	153	 
HVP	6 845(40)	+ others
HLbL	92(17)	  
total	116 591 810(43)	[g - 2 white paper 20]
FNAL+BNL	116 592 062(40)	

largest source of uncertainty & non-perturbative



this problem is bigger than $g - 2$! [CMD-3 23] [BMW 20]

largest source of uncertainty & non-perturbative



this problem is bigger than $g - 2$! [CMD-3 23] [BMW 20]

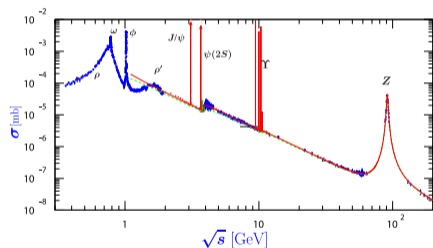
using optical theorem $s > 0$

- measure $ee \rightarrow \text{hadrons}$
- remove radiative corrections
- extrapolate to $s \rightarrow \infty$ using pQCD
- integrate over s

$$\int ds \left(K(s) \left[\text{Diagram: } ee \rightarrow \text{hadrons} \right] \right)$$

The diagram shows an electron-positron annihilation process into a hadronic final state, represented by a semi-circle.

- 72% (78%) of value (uncertainty) from the $ee \rightarrow \pi\pi$ channel $s \lesssim 1 \text{ GeV}$

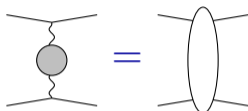


measure low Q^2 regions

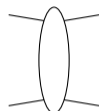
- instead measure in t -channel, i.e. space-like
- no resonances \rightarrow much cleaner signal
- HVP is loop-induced \rightarrow much smaller signal ($10^{-3} \times \text{LO}$)
- competitive extraction @ 10^{-2}

\Rightarrow goal for MUonE: measure $e\mu \rightarrow e\mu$ @ 10^{-5}

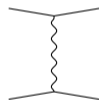
$$\int dt \left(K'(t) \text{ [diagram of a loop with a shaded blob] } \right) \quad [\text{MUonE 19}]$$



=



—



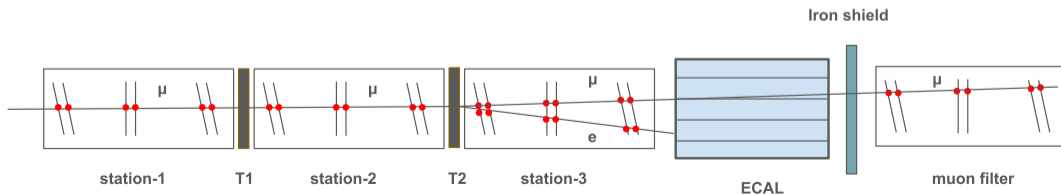
textbook QED

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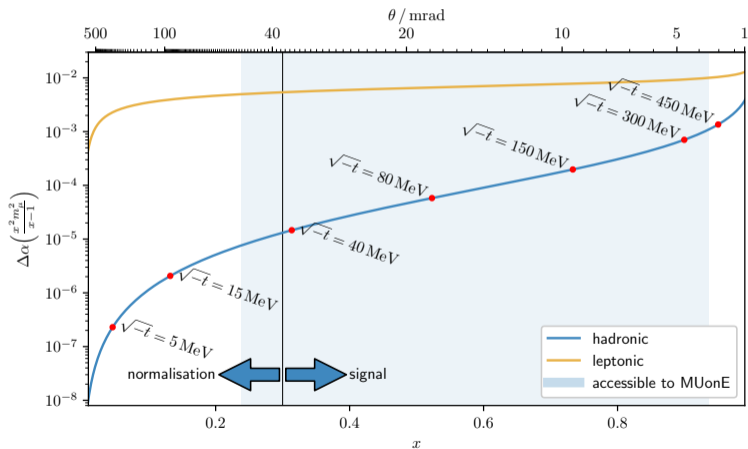
QED

5+ years,
4+ workshops,
34+ authors

- scattering μ of low- Z material (${}_4\text{Be}$)
- pure t -channel $-s \simeq Q^2 \simeq 0$
- \Rightarrow high $s \leftrightarrow$ measure more of the curve
- beam energy needs to be quite high $E_\mu \simeq 160 \text{ GeV}$
- \Rightarrow M2 muon beam at CERN North Area
- main measurement: θ_e, θ_μ
 - + E_{beam} for calibration
 - + E_μ for particle ID



cancel systematic effects $\left(\frac{d\sigma}{d\theta}\right)_{\text{sig}} / \left(\frac{d\sigma}{d\theta}\right)_{\text{norm}}$



6 MUonE (adjacent) theory workshops over 6+ years



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	problem	solution	what?	doable up to?
①	lots of masses	massification	expand in m_e^2/Q^2	LP, three-loop
②	numerical issues in real corrections	NTS stabilisation jettification	expand in $E_\gamma/\sqrt{Q^2}$ expand in $\cos\theta \rightarrow 1$	NLP, all-orders LP, one-loop
③	phase space	FKS ^ℓ	YFS-inspired subtraction scheme	all-orders

- NNLO double-boxes: ①
- NNLO real-virtual: ②
- N³LO real-virtual-virtual: ①, ②, jettification

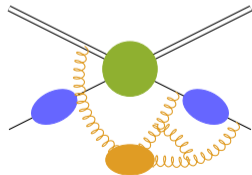


masses are physical in QED \Rightarrow keep masses

- drop polynomially suppressed terms at two-loop \rightarrow error $\sim \left(\frac{\alpha}{\pi}\right)^2 \log \frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$
- based on factorisation, SCET, and method of regions
[Penin 06; Mitov, Moch 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g. $e\mu \rightarrow e\mu$ at two-loop:

$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$

- **soft**: process-dependent $\mathcal{S} = 1 + \text{fermion loops}$
 \rightarrow compute separately anyway to combine with hadron loops
- **collinear**: universal Z , converts $1/\epsilon \rightarrow \log(m^2/Q^2)$
- **hard**: massless calculation



real-virtual corrections trivial in principle, extremely delicate numerically

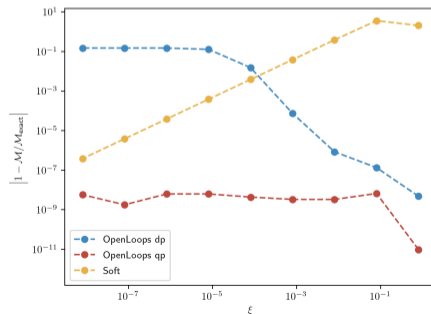


$$\text{Diagram} = \frac{1}{E_\gamma^2} \underbrace{\text{Diagram}}_{\text{eikonal}} + \mathcal{O}(E_\gamma^{-1})$$

example $ee \rightarrow ee\gamma$

[Engel, Signer, YU 21; Kollatzsch, YU 22; Engel 23]

- soft limit (of collinear emission)
 $E_\gamma = \xi \sqrt{s}/2$
- arbitrary prec. calculation vs **dp**, **qp**,
eikonal, **NTS**
- stability problem solved & speed-up



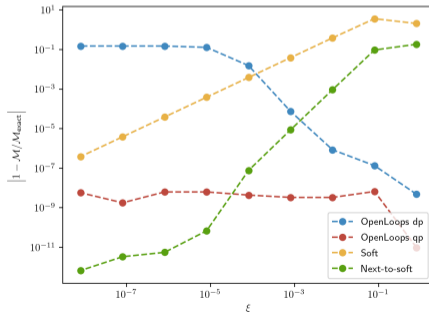
real-virtual corrections trivial in principle, extremely delicate numerically

$$\text{Diagram} = \frac{1}{E_\gamma^2} \underbrace{\mathcal{E} \text{ Diagram}}_{\text{eikonal}} + \frac{1}{E_\gamma} \left\{ \underbrace{D \left[\text{Diagram} \right]}_{\text{LBK}} + \underbrace{\mathcal{S} \text{ Diagram}}_{\text{soft function}} \right\} + \mathcal{O}(E_\gamma^0)$$

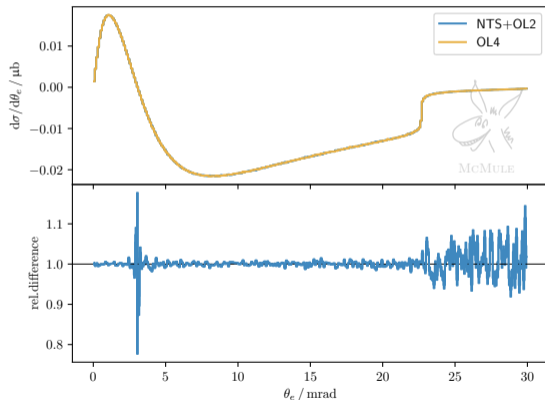
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test next-to-soft stabilisation vs OL4 (OpenLoops quad) for $\mu e \rightarrow \mu e$ real-virtual



- same statistics, same result
 - 70 days vs 4 days
 - integrated results for different cuts
- ⇒ this is **not** an approximation but a numerical tool

NTS	OL4
-0.29268(4)	-0.29267(4)
-0.44789(6)	-0.44778(6)
-0.64662(9)	-0.64649(9)

- universal soft limit $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E} \mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f} = \text{finite}$

use this to construct an all-order subtraction scheme FKS^ℓ [Engel, Signer, YU 19]

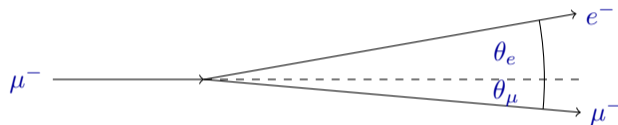
- nothing complicated needed higher than $\mathcal{O}(\epsilon^0)$
- only one universal CT: $\hat{\mathcal{E}}$

$$\underbrace{\int d\Phi_\gamma \text{ (grey blob) }}_{\text{divergent and complicated}} = \underbrace{\int d\Phi_\gamma \left(\text{grey blob} - \text{green blob} \right)}_{\text{complicated but finite}} + \underbrace{\int d\Phi_\gamma \text{ (green blob) }}_{\text{divergent but easy}}$$

implemented in McMULE v0.4.2

<https://mule-tools.gitlab.io>

- $\mu^- e^- \rightarrow \mu^- e^-$

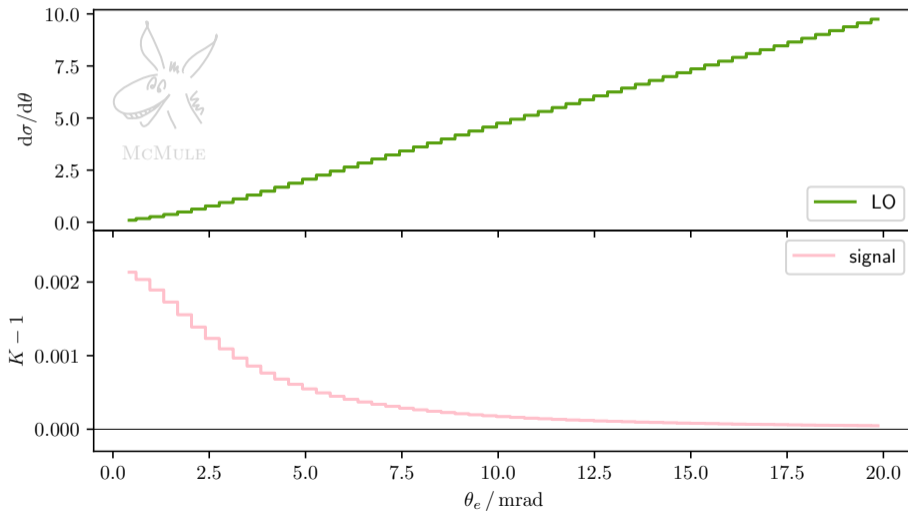


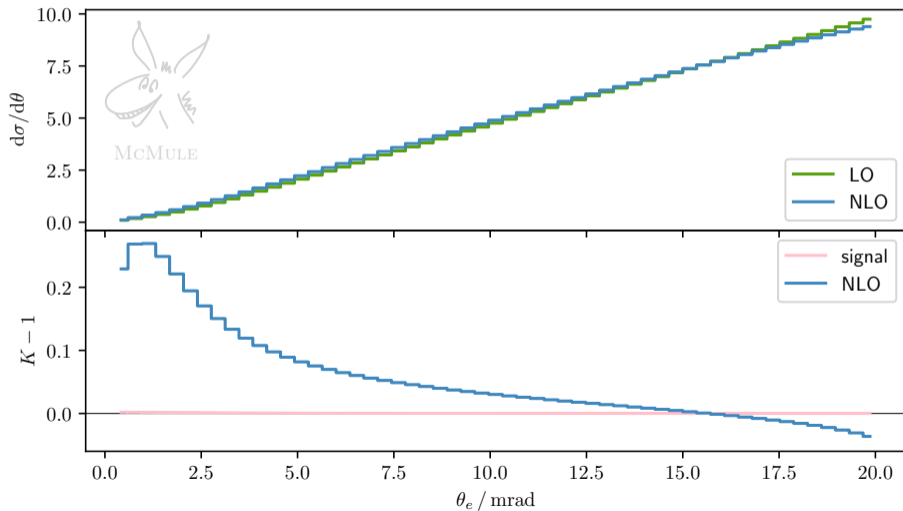
- S1: $E_e > 1 \text{ GeV}$, $\theta_\mu > 0.3 \text{ mrad}$
- run for 2.5 CPU yr
(290 kWh energy / 3.5 kgCO₂e)

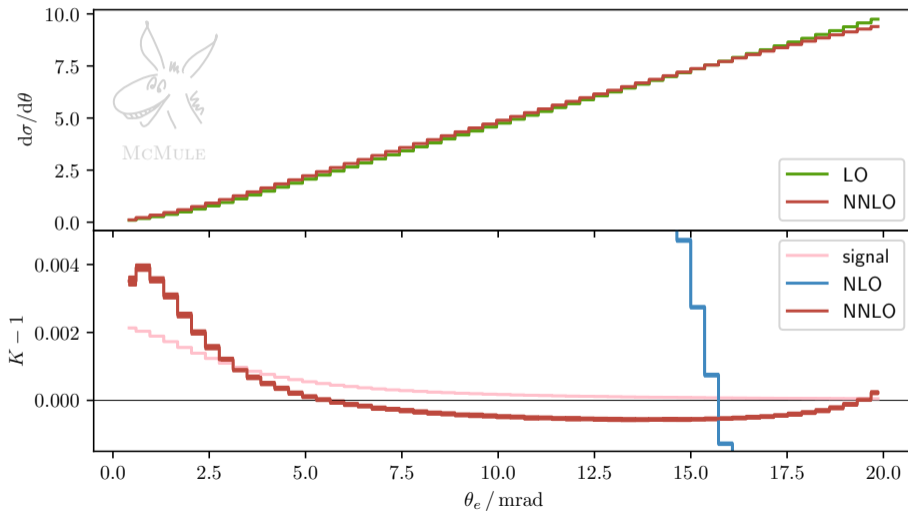


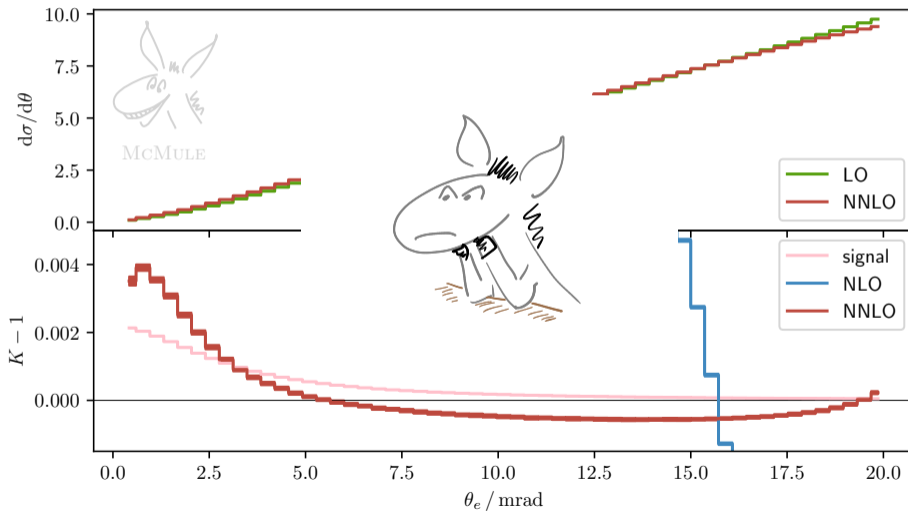
[Broggio, Engel, Ferrogli, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]

all results and data: <https://mule-tools.gitlab.io/user-library/mu-e-scattering/muone-full-legacy/>



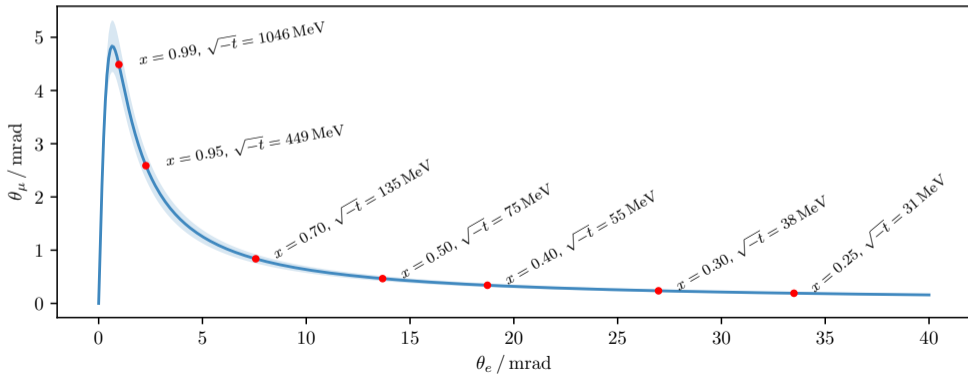


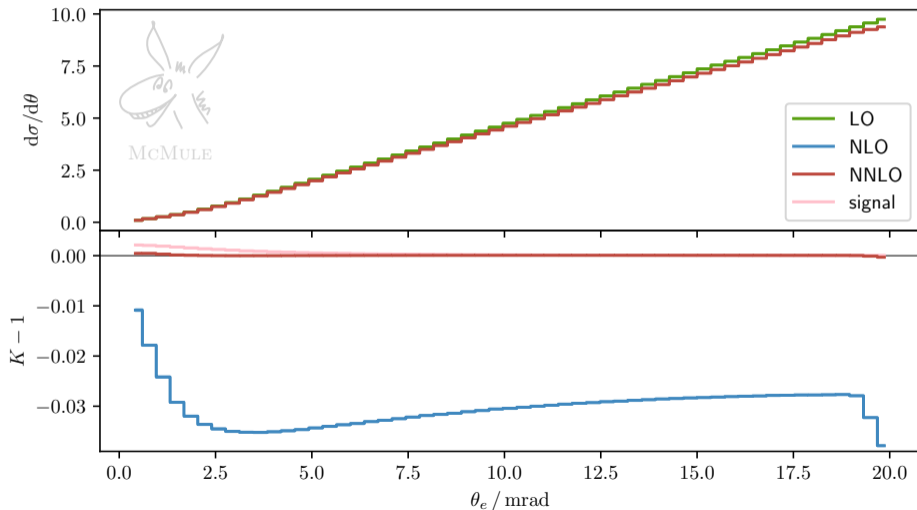


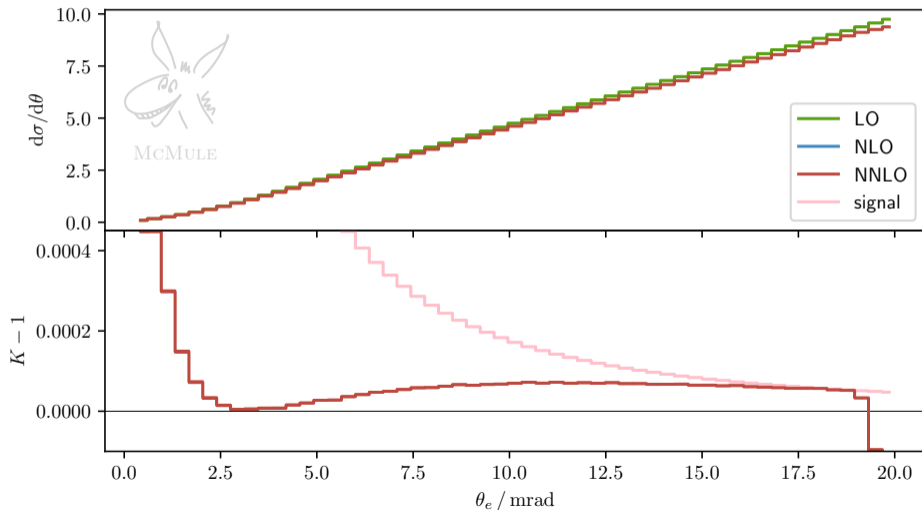


this clearly isn't working

- at this rate ($\sim 10\%$ NLO, $\sim 0.1\%$ NNLO), we would need N⁴LO to reach 10^{-5}
- most of this is due to hard radiation
- S2: same as S1 + needs to be in the band







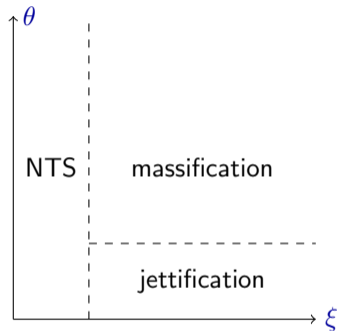
$ee \rightarrow \gamma^*$ can be taken to N³LO

- VVV: known
[Fael, Lange, Schönwald, Steinhauser 22]
- RRR: “trivial”
- RRV: OpenLoops + NTS stabilisation
- RVV: massless (three-jet @ NNLO)
[Badger, Kryś, Moodie, Zoia 23]

⇒ LBK + jettification at two-loop

jettification

- expand for small emission angles




expand for $m_e^2 \sim p_e \cdot p_\gamma \ll p_e \cdot q \sim p_\gamma \cdot q$

- calculation in SCET
- two non-trivial scales: $(p_e \cdot p_\gamma)/m_e^2$ and $(p_e \cdot q)/(p_\gamma \cdot q)$
- integrals not regularised in DIMREG

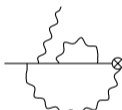
$$\frac{1}{\ell \cdot \bar{n}} \rightarrow \frac{1}{(\ell \cdot \bar{n})^{1+\eta}}$$

or

$$\frac{1}{\ell \cdot \bar{n}} \rightarrow \frac{1}{\ell \cdot \bar{n} + \Delta}$$




$$\sim \int \frac{1}{\ell_1 \cdot \bar{n}} \frac{1}{\ell_1^2} \frac{1}{\ell_2 \cdot \bar{n}} \frac{1}{\ell_2^2} \dots$$



$$\sim \int \frac{1}{\ell_1 \cdot \bar{n}} \frac{1}{\ell_1^2} \frac{1}{\ell_2^2} \dots$$

- either complicates the integrals
- final result J finite in η or Δ



$$\sim \int \frac{1}{\ell_1^2} \frac{1}{\ell_2^2} \dots$$

[WIP, Schalch, Engel, YU]

- ✓ first NNLO with multiple external masses
[Broggio, Engel, Ferroglia, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]
- ✓ event generation (not in McMULE)
- ✓ iterative HVP extraction procedure
[Fael 18]
- ✓ precision now: $\mathcal{O}(10^{\{-3,-4\}})$, goal: $\mathcal{O}(10^{-5})$
 - lots of optimisation still possible
(observable, μ^+ vs. μ^- beam, polarisation etc)
 - resummation (analytic & parton shower)
 - partial N³LO ($Q_e^8 Q_\mu^2$)





f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI),
S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg),
V.Sharkovska (Zurich & PSI), Y.Ulrich (Bern), A.Gurgone (Pavia)
not pictured: P.Banerjee (IIT Guwahati), D.Moreno (PSI), D.Radic (PSI)



McMule

mule-tools.gitlab.io

the beam can do both μ^+ and μ^-

$$\begin{aligned}
 \sigma \sim Q_e Q_\mu & \left(Q_e^2 Q_\mu^1 \times \text{[diagram: photon exchange]} \right. \\
 & + \underbrace{Q_e^3 Q_\mu^1}_{\text{easy}} \times \text{[diagram: photon exchange with electron loop]} + \underbrace{Q_e^2 Q_\mu^2}_{\text{okay}} \times \text{[diagram: photon exchange with muon loop]} + \underbrace{Q_e^1 Q_\mu^3}_{\text{easy}} \times \text{[diagram: photon exchange with electron loop]} \\
 & + \underbrace{Q_e^5 Q_\mu^1}_{\text{easy}} \times \text{[diagram: photon exchange with electron loop]} + \underbrace{Q_e^4 Q_\mu^2}_{\text{really difficult}} \times \text{[diagram: photon exchange with muon loop]} + \underbrace{Q_e^3 Q_\mu^3}_{\text{really difficult}} \times \text{[diagram: photon exchange with muon loop]} \\
 & + \underbrace{Q_e^2 Q_\mu^4}_{\text{really difficult}} \times \text{[diagram: photon exchange with muon loop]} + \underbrace{Q_e^1 Q_\mu^5}_{\text{easy}} \times \text{[diagram: photon exchange with muon loop]} \left. \right)
 \end{aligned}$$

- proposal $\sigma(\mu^+) + \sigma(\mu^-)$

⇒ some of the difficult stuff cancels

