

**MEP Seminar**

# **QED with McMULE for PRad**

**Yannick Ulrich**

**for the McMULE team**

**AEC, University of Bern**

**3 APRIL 2024**

fixed-order NNLO QED framework Monte Carlo for MUons and other LEptons

- provided: matrix elements by us or others
- output: **physical cross section** for any physical observable
- McMULE: phase space generation, subtraction, stabilisation, integration, event generation, etc.
- all leptonic  $2 \rightarrow 2$  processes in QED at NNLO (+ a few others)
- stable public version is an integrator
- generator on development branch

Get the code here: <https://mule-tools.gitlab.io>

Read the docs here: <https://mcmule.readthedocs.io>

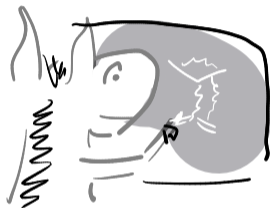


McMULE

process	experiment	physics motivation	order
$e\mu \rightarrow e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$lp \rightarrow lp$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO
$eN \rightarrow eN$	PRad, ULQ2	background	+
$e^-e^- \rightarrow e^-e^-$	Prad 2	normalisation	NNLO
	MOLLER, ...	$\sin^2 \theta_W$ at low $Q^2$	
$e^+e^- \rightarrow e^+e^-$	any $e^+e^-$ collider	luminosity measurement	NNLO
$ee \rightarrow ll$	VEPP, BES, Daphne, ...	$R$ -ratio	NNLO±
	Belle	$\tau$ properties	
$ee \rightarrow \gamma\gamma$	Daphne	dark searches	NNLO-
	any $e^+e^-$ collider	luminosity measurement	
$e\nu \rightarrow e\nu$	DUNE	flux & $\sin^2 \theta_W$	NNLO-
$\mu \rightarrow \nu\bar{\nu}e$	MEG	ALP searches	NNLO+
	DUNE	beam-line profiling	
$\mu \rightarrow \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \rightarrow \nu\bar{\nu}eee$	MEG, Mu3e	background	NLO
$ee \rightarrow \pi\pi$	VEPP, BES, Daphne, ...	$R$ -ratio	+
$ee \rightarrow ll\gamma$	VEPP, BES, Daphne, ...	$R$ -ratio	+

process	experiment	physics motivation	order
$e\mu \rightarrow e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$lp \rightarrow lp$	P2, Muse, Prad. QWeak. ...	proton radius and weak charge	NNLO
$eN \rightarrow eN$	PRad, ULQ2		+
$e^-e^- \rightarrow e^-e^-$	Prad 2	on	NNLO
$e^+e^- \rightarrow e^+e^-$	MOLLER, ...	low $Q^2$	
$ee \rightarrow ll$	any $e^+e^-$ col	measurement	NNLO
$ee \rightarrow \gamma\gamma$	VEPP, BES, Belle	s	NNLO±
$ee \rightarrow \nu\bar{\nu}$	Daphne	es	NNLO-
$e\nu \rightarrow e\nu$	any $e^+e^-$ col	measurement	
$\mu \rightarrow \nu\bar{\nu}e$	DUNE	$\theta_W$	NNLO-
$\mu \rightarrow \nu\bar{\nu}ee$	MEG		NNLO+
	DUNE	goal: world domination filing	
$\mu \rightarrow \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \rightarrow \nu\bar{\nu}eee$	MEG, Mu3e	background	NLO
$ee \rightarrow \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \rightarrow ll\gamma$	VEPP, BES, Daphne, ...	R-ratio	+





theory background

$$\begin{aligned}
 \sigma &= \int d\Phi_2 \left| \begin{array}{c} \text{tree} \\ \text{one-loop} \\ \text{two-loop} \\ \text{three-loop} \\ \dots \end{array} \right|^2 \\
 &+ \int d\Phi_3 \left| \begin{array}{c} \text{one-loop} \\ \text{two-loop} \\ \text{three-loop} \\ \dots \end{array} \right|^2 \\
 &+ \int d\Phi_4 \left| \begin{array}{c} \text{two-loop} \\ \text{three-loop} \\ \dots \end{array} \right|^2 \\
 &+ \int d\Phi_5 \left| \begin{array}{c} \text{three-loop} \\ \dots \end{array} \right|^2 \\
 &+ \dots
 \end{aligned}$$

## challenges to overcome

- divergent phase space integration
- ⇒ FKS<sup>ℓ</sup>
- numerical instabilities
- ⇒ next-to-soft stabilisation
- virtual amplitudes with  $m \neq 0$
- ⇒ OpenLoops (one-loop) massification (two-loop)
- negative weights
- ⇒ cell resampling

subtract universal counter term from divergent real correction

$$\int d\Phi_\gamma \underbrace{\text{diagram}}_{\propto E_\gamma^{-2}} = \underbrace{\int d\Phi_\gamma \left( \text{diagram} - \text{diagram} \right)}_{\text{complicated but finite}} + \underbrace{\int d\Phi_\gamma \text{diagram}}_{\text{divergent but easy}}$$

- works to all order in QED [Engel, Signer, YU 19]
- no resolution parameter  $\omega_c$
- unphysical & arbitrary  $0 < \xi_c \lesssim 1$
- singularities are treated locally  $\rightarrow$  stable numerical integration

real-virtual corrections trivial in principle, delicate in practise

$$\text{Diagram} \xrightarrow{E_\gamma \rightarrow 0} \underbrace{\frac{1}{E_\gamma^2} \text{Diagram}}_{\text{eikonal}} + \underbrace{\frac{1}{E_\gamma} \text{Diagram}}_{\text{next-to-soft}} + \mathcal{O}(E_\gamma^0)$$

- based on LBK theorem [Low 58; Burnett, Kroll 67] and extensions [Engel, Signer, YU 21; Engel 23]
- if  $E_\gamma < E_{\text{NTS}} \approx 10^{-3} \sqrt{s}/2$ , switch to NTS expansion rather than full expression
- introduces small theory error  $\mathcal{O}(10^{-3}) \times \sigma^{(2)} = \mathcal{O}(10^{-6})$   
 $\Rightarrow$  well below the N<sup>3</sup>LO
- significant speed-up: 7 days vs. 3 months

two-loop integrals with masses are really difficult

- but  $m_\ell^2 \ll m_p^2 \sim s \sim Q^2$
- expand in  $m_\ell^2/Q^2$

$$\text{Diagram} \sim A \log^2 \frac{m_\ell^2}{Q^2} + B \log \frac{m_\ell^2}{Q^2} + C + \mathcal{O}\left(\frac{m_\ell^2}{Q^2}\right)$$

- can be done easily by using  $m_\ell = 0$  result up to three-loop  
 [Penin 06; Becher, Melnikov 07; Engel, Gneidiger, Signer, YU 18; YU 23]
- introduces small theory error  $\mathcal{O}(10^{-2}) \times \sigma^{(2)} = \mathcal{O}(10^{-5})$   
 $\Rightarrow$  well below statistical error

## all of this was for an integrator

- calculate arbitrary differential distributions
  - event generation by just dumping momenta to file (“garden hose approach”)
  - if  $r \times N$  of  $N$  weights are negative, we need  $\propto 1/(1 - 2r)^2$  events
- ⇒ reduce  $r$  as much as possible by cancelling negative weights as early as possible
- optimisations from splitting integrand goes away



... at NLO for simplicity

$$\sigma_{\text{NLO}} = \int \text{[tree]} + \frac{\alpha}{4\pi} \int \text{[1-loop]} + \frac{\alpha}{4\pi} \int \text{[2-loop]}$$

- slicing: fairly few negative weights **but** numerically construct  $\log \omega_c$

$$= \int \underbrace{\left( \text{[tree]} + \frac{\alpha}{4\pi} \text{[1-loop]} + \frac{\alpha}{4\pi} \int_1 \text{[2-loop]} \right)}_{\text{mostly } > 0} + \frac{\alpha}{4\pi} \int_{\omega > \omega_c} \underbrace{\text{[1-loop]}}_{> 0}$$

- subtraction: easier integration **but** lots and lots of negative weights ( $\mathcal{O}(5\%)$  at NLO, more at NNLO)

$$= \int \underbrace{\left( \text{[tree]} + \frac{\alpha}{4\pi} \text{[1-loop]} + \frac{\alpha}{4\pi} \int_1 \text{[2-loop]} \right)}_{\text{mostly } > 0} + \frac{\alpha}{4\pi} \int \underbrace{\left( \text{[1-loop]} - \text{[2-loop]} \right)}_{\text{whatever}}$$

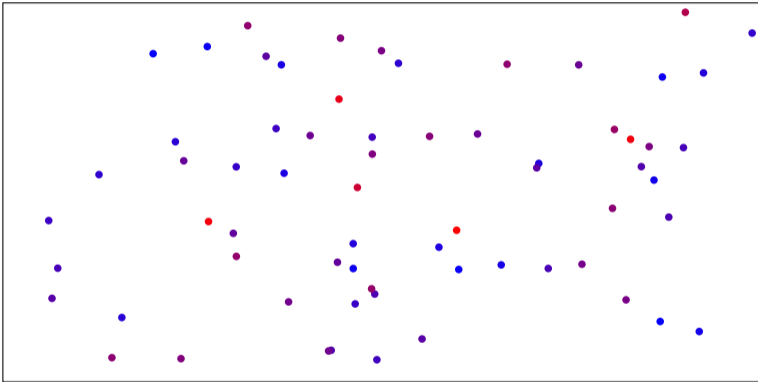
## two observations

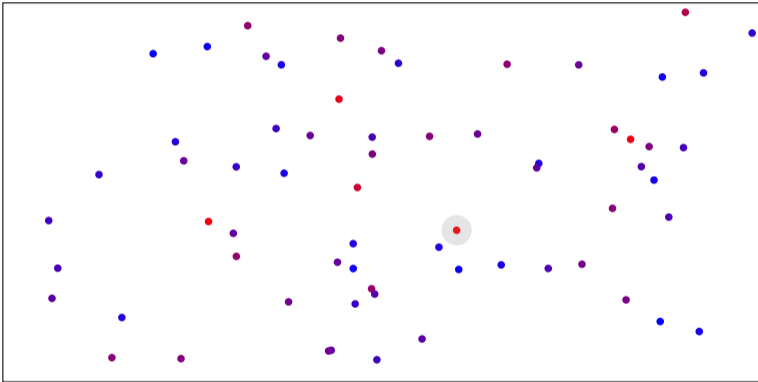
- ① cross section  $\sigma = \int_{\mathcal{C}} d\sigma > 0$ , irregardless of the size of integration region  $\mathcal{C}$
- ② experiments have a finite resolution  
(we already knew that because we can't see soft photons)

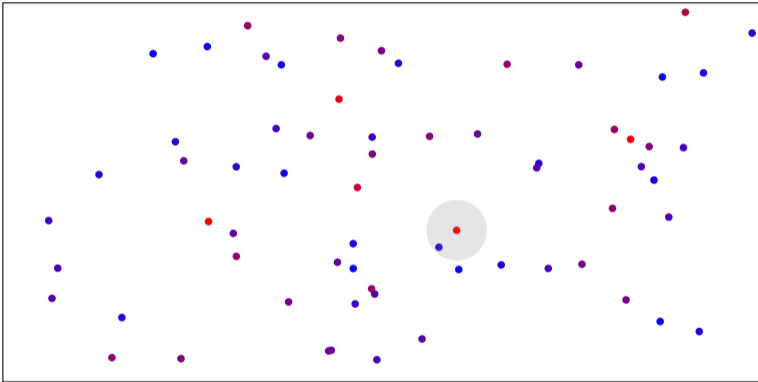
## algorithm to remove negative weights [Andersen, Maier 21]

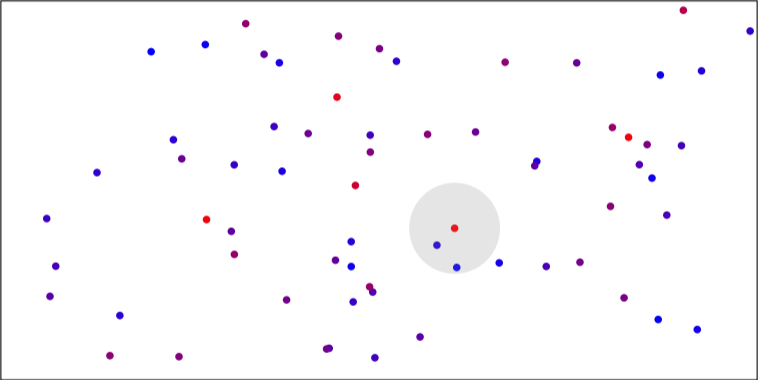
- pick an event with  $w_i < 0$
- find nearby events until  $\sum_{i \in \mathcal{C}} w_i > 0$
- if  $\mathcal{C}$  gets too big (events become resolvable), abort (or add more events)
- else  $w_i \rightarrow \frac{\sum_{j \in \mathcal{C}} w_j}{\sum_{j \in \mathcal{C}} |w_j|} w_i$

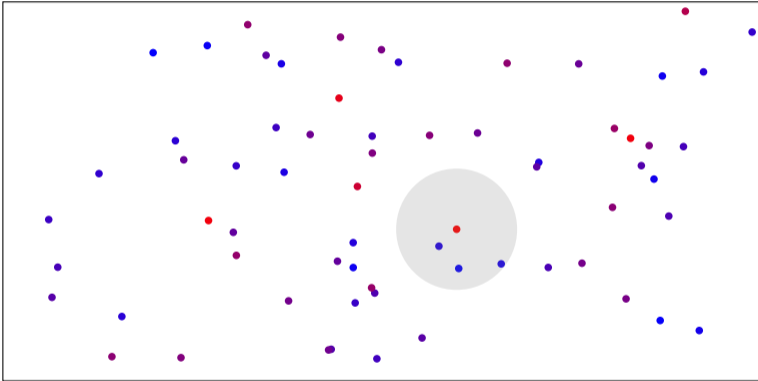
we can remove negative weights without biasing physical observables!

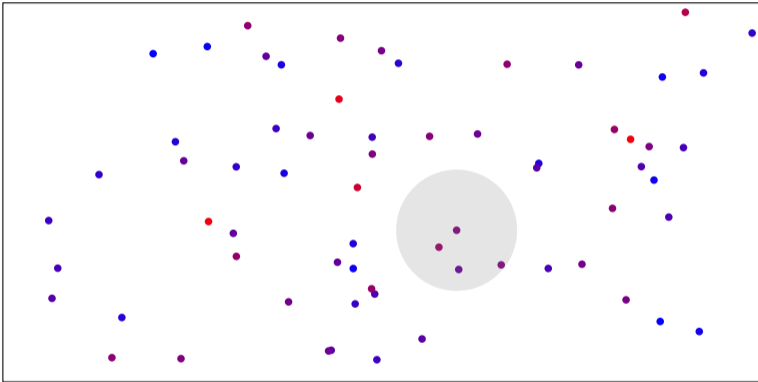


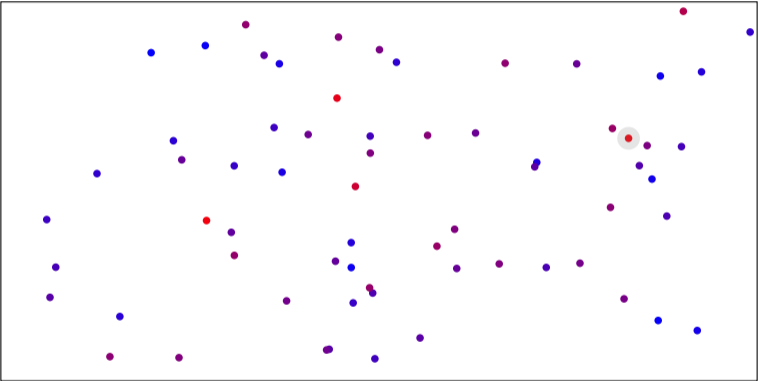


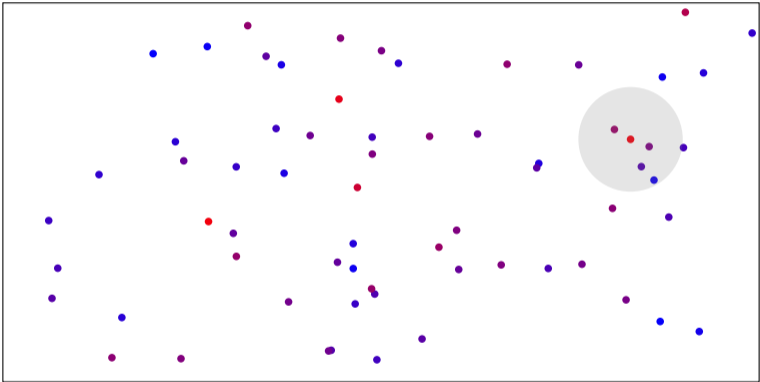


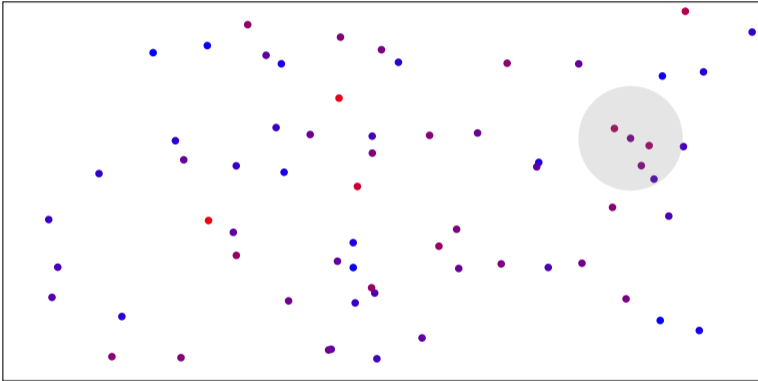


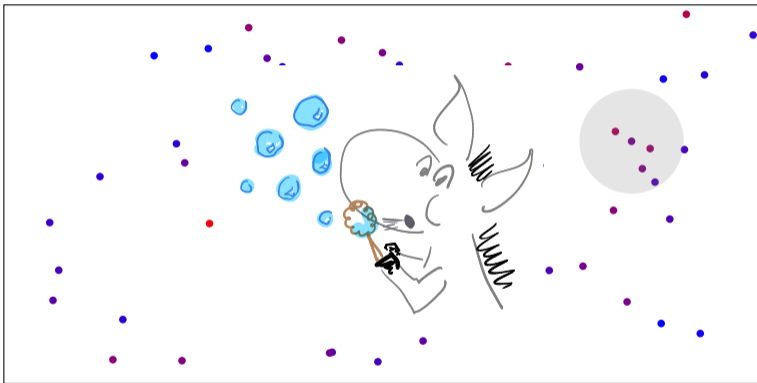






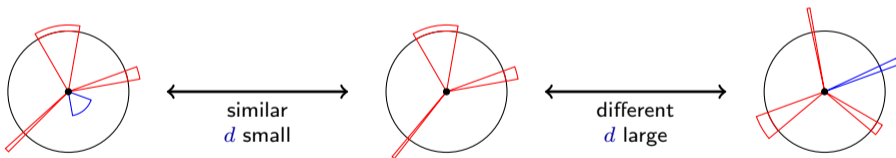






we need to define a metric in event space  $d(e_1, e_2) \geq 0$

- doesn't really matter how we do this as long as IR safe  
(events with soft photons are near each other)
- ideally: events that look similar are closer to each other than those that don't



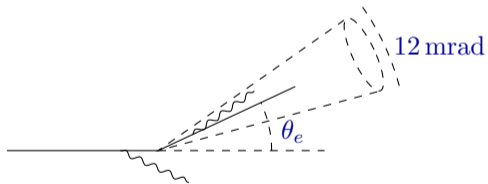


results for PRad

$$E_{\text{beam}} = 1.1 \text{ GeV}$$

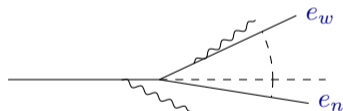
$e$ - $p$ :

- $0.7^\circ < \theta_e < 6.0^\circ$
- no more than 20 MeV photons outside a 12 mrad cone around the electron



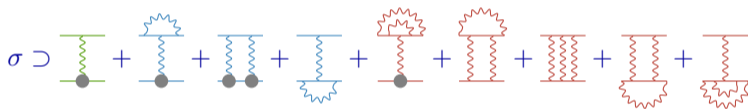
$e$ - $e$ :

- $0.5^\circ < \theta_e < 6.5^\circ$
- $\sum E_\gamma < 131.95 \text{ MeV}$ ,  
 $|\mathbf{180}^\circ - |\phi_n - \phi_w|| < 7.35^\circ$

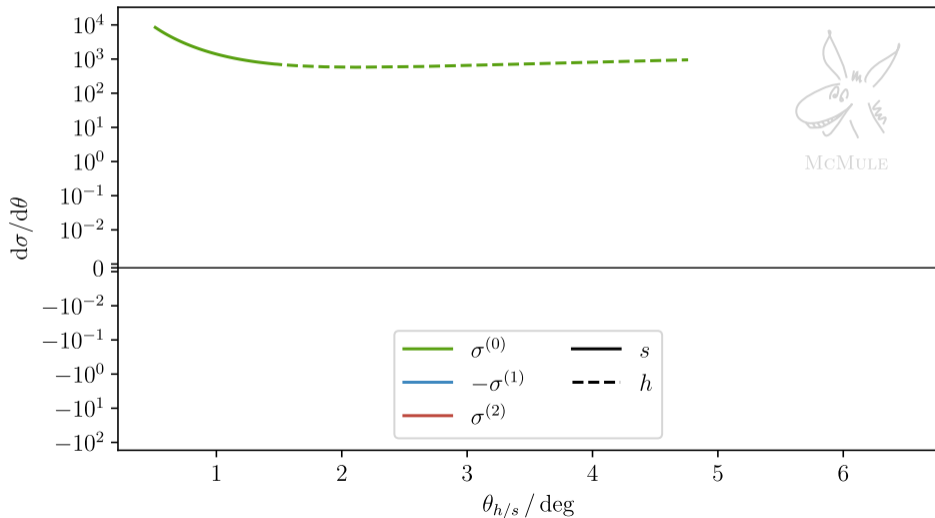


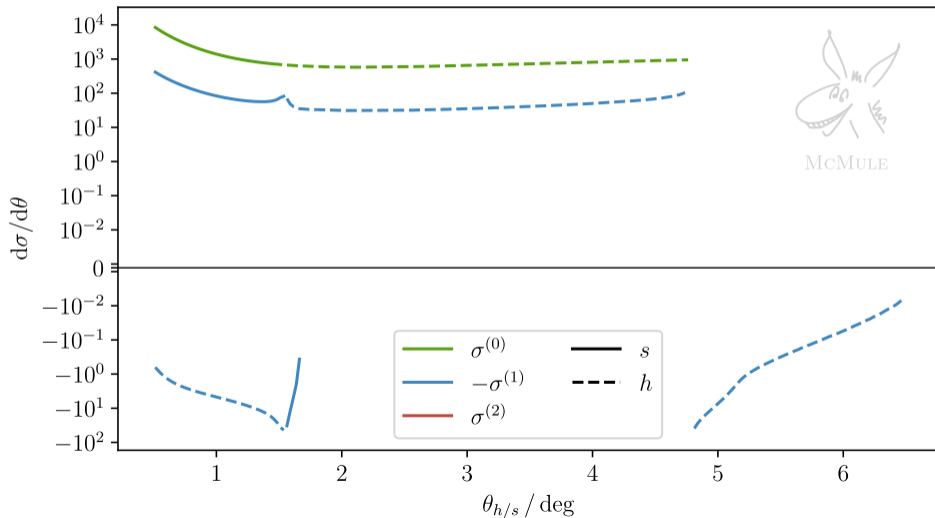
not to scale

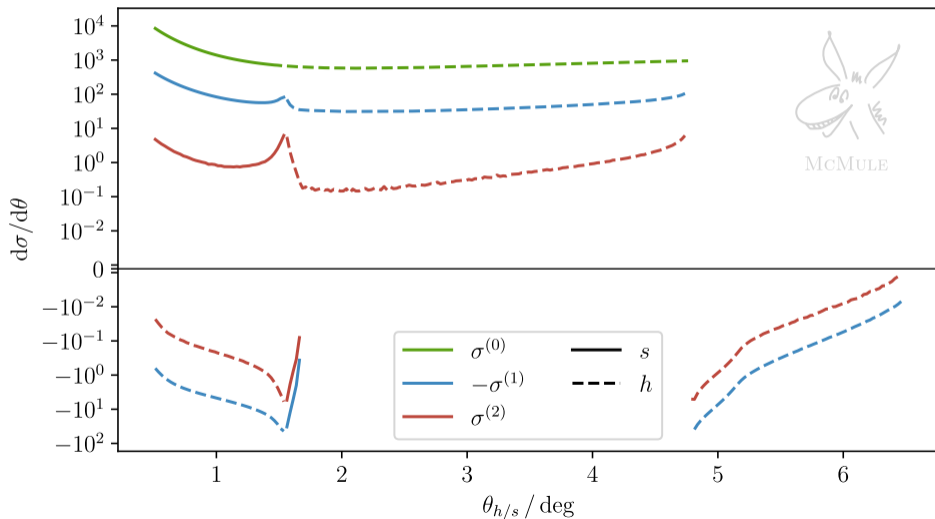
- simple dipole model for proton form factor  $G_E = \frac{G_M}{1+\kappa} = \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2}$

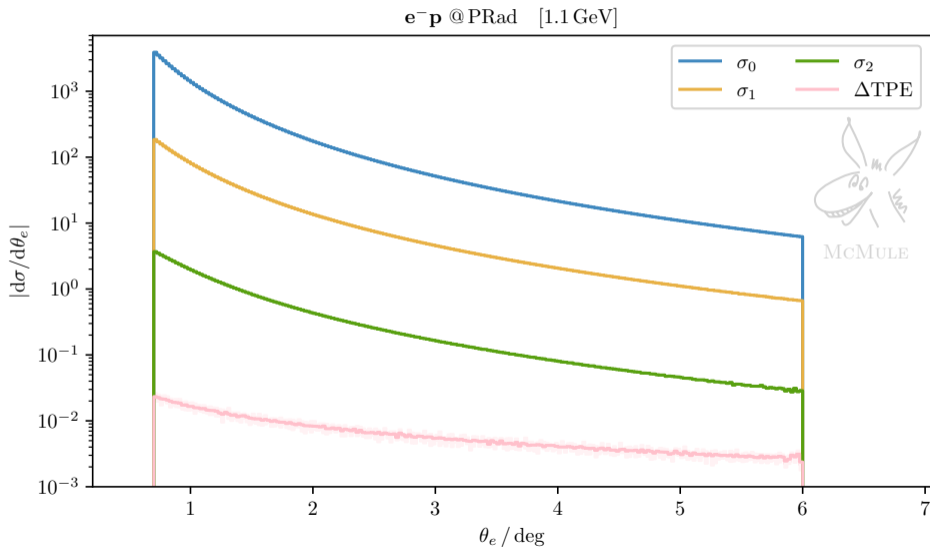


- study TPE effects with  $\Delta\text{TPE} = \left[ \text{diagram}(\Lambda^2) - \text{diagram} \right]_{\Lambda^2 = \{0.88, 0.71, 0.60\} \text{ GeV}^2}$









## point-like calculations

- approximate NNLO for  $ep \rightarrow ep\gamma$  from  $pp \rightarrow 2j + \gamma$   
[Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia 23]
- full mass dependence plausible but **very** difficult

## form-factor calculations

- partial N<sup>3</sup>LO for  $ep \rightarrow ep$  (and who knows, maybe  $ee \rightarrow ee$ )
- various performance improvements

## better hadronic models

- TPE: WIP with Marc Vanderhaeghen
- inelastic: interest from the EIC community?

## general

- resummation of soft photon emission



## McMULE is a state-of-the-art framework for QED

- we have the best QED calculation of pointlike  $ep \rightarrow ep$  and  $ee \rightarrow ee$
- it **will** get better
- we have an okay model of TPE
- it **will** get better
- we could add inelastic effects for EIC
- **please** get in touch so we can help you use McMULE!  
[yannick.ulrich@cern.ch](mailto:yannick.ulrich@cern.ch)





f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI),  
S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg),  
V.Sharkovska (Zurich & PSI), Y.Ulrich (Bern), A.Gurgone (Pavia)  
not shown: P.Banerjee (IIT Guwahati), D. Moreno (PSI), D. Radic (PSI)



McMULE

[mule-tools.gitlab.io](https://mule-tools.gitlab.io)

