CFNS Seminar

QED with McMule for MUSE

Yannick Ulrich

for the McMule team

AEC, University of Bern

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what is McMule?

fixed-order NNLO QED framework Monte Carlo for MUons and other LEptons

- provided: matrix elements by us or others
- output: physical cross section for any physical observable
- McMule: phase space generation, subtraction, stabilisation, integration, event generation, etc.
- all leptonic $2 \to 2$ processes in QED at NNLO (+ a few others)
- stable public version is an integrator
- generator on development branch

Get the code here: `https://mule-tools.gitlab.io`
Read the docs here: `https://mcmule.readthedocs.io`

McMule

process	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$\ell p \to \ell p$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO
$eN \to eN$	PRad, ULQ2	background	+
$e^-e^- \to e^-e^-$	Prad 2	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \ell\ell$	VEPP, BES, Daphne, ...	R-ratio	NNLO±
	Belle	τ properties	
$ee \to \gamma\gamma$	Daphne	dark searches	NNLO−
	any e^+e^- collider	luminosity measurement	
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO−
$\mu \to \nu\bar\nu e$	MEG	ALP searches	NNLO+
	DUNE	beam-line profiling	
$\mu \to \nu\bar\nu e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \to \nu\bar\nu eee$	MEG, Mu3e	background	NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	+

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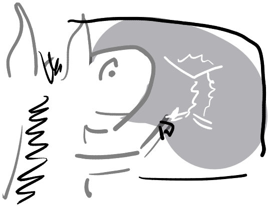
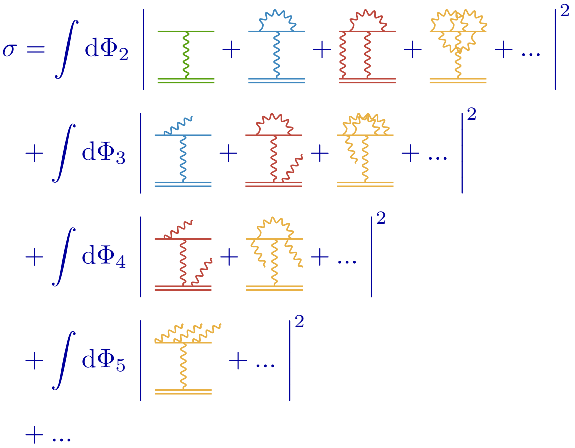
$e^-e^- \to e^-e^-$	Prad 2	on	NNLO
	MOLLER, ...	low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- col	measurement	NNLO
$ee \to \ell\ell$	VEPP, BES,		NNLO±
	Belle	s	
$ee \to \gamma\gamma$	Daphne	es	NNLO−
	any e^+e^- col	measurement	
$e\nu \to e\nu$	DUNE	θ_W	NNLO−
$\mu \to \nu\bar{\nu}e$	MEG		NNLO+
	DUNE	filing	
$\mu \to \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \to \nu\bar{\nu}eee$	MEG, Mu3e	background	NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	+

goal: world domination



theory background



challenges to overcome

- divergent phase space integration

⇒ FKS$^\ell$

- numerical instabilities

⇒ next-to-soft stabilisation

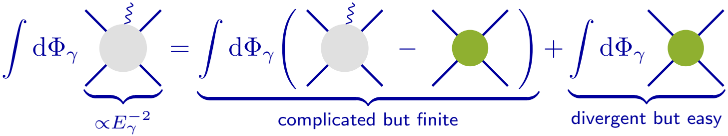
- virtual amplitudes with $m \neq 0$

⇒ OpenLoops (one-loop) massification (two-loop)

- negative weights

⇒ cell resampling

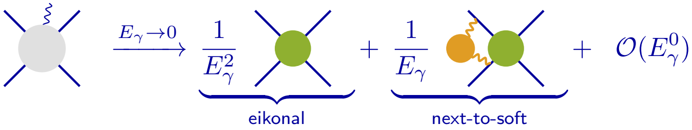
subtract universal counter term from divergent real correction



- works to all order in QED [Engel, Signer, YU 19]
- no resolution parameter ω_c
- unphysical & arbitrary $0 < \xi_c \lesssim 1$
- singularities are treated locally $\rightarrow$ stable numerical integration

next-to-soft stabilisation

real-virtual corrections trivial in principle, delicate in practise

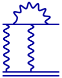


- based on LBK theorem [Low 58; Burnett, Kroll 67] and extensions [Engel, Signer, YU 21; Engel 23]
- if $E_\gamma < E_{\text{NTS}} \approx 10^{-3}\sqrt{s}/2$, switch to NTS expansion rather than full expression
- introduces small theory error $\mathcal{O}(10^{-3}) \times \sigma^{(2)} = \mathcal{O}(10^{-6})$
  $\Rightarrow$ well below the N^{3}LO
- significant speed-up: 7 days vs. 3 months

massification

two-loop integrals with masses are really difficult

- but $m_\ell^2 \ll m_p^2 \sim s \sim Q^2$
- expand in m_ℓ^2/Q^2



$$\sim A \log^2 \frac{m_\ell^2}{Q^2} + B \log \frac{m_\ell^2}{Q^2} + C + \mathcal{O}\Big(\frac{m_\ell^2}{Q^2}\Big)$$

- can be done easily by using $m_\ell = 0$ result up to three-loop
  [Penin 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18; YU 23]
- introduces small theory error $\mathcal{O}(10^{-2}) \times \sigma^{(2)} = \mathcal{O}(10^{-5})$
  $\Rightarrow$ well below statistical error

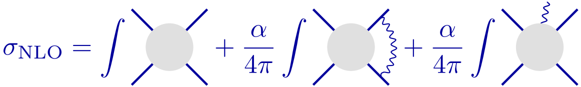
all of this was for an integrator

- calculate arbitrary differential distributions
- event generation by just dumping momenta to file ("garden hose approach")
- if $r \times N$ of N weights are negative, we need $\propto 1/(1-2r)^2$ events

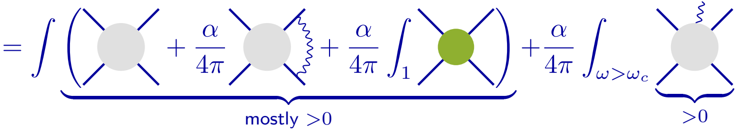
$\Rightarrow$ reduce r as much as possible by cancelling negative weights as early as possible

- optimisations from splitting integrand goes away

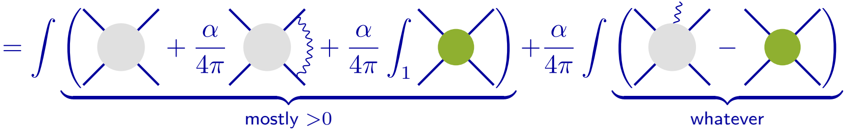
... at NLO for simplicity



- slicing: fairly few negative weights but numerically construct $\log \omega_c$



- subtraction: easier integration but lots and lots of negative weights ($\mathcal{O}(5\%)$ at NLO, more at NNLO)



can we get both?

two observations

1. cross section $\sigma = \int_{\mathcal{C}} \mathrm{d}\sigma > 0$, irregardless of the size of integration region $\mathcal{C}$
2. experiments have a finite resolution

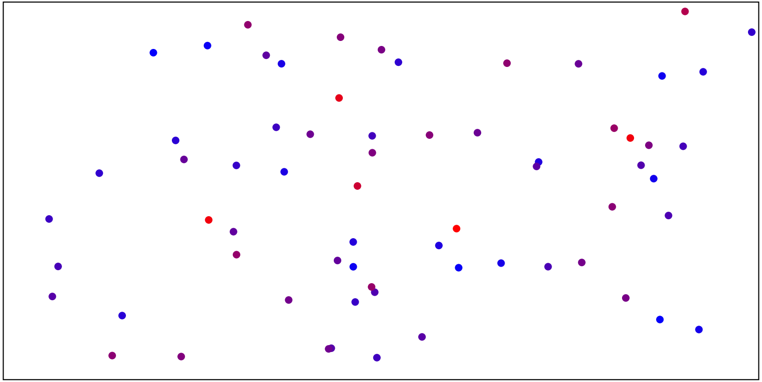
   (we already knew that because we can't see soft photons)

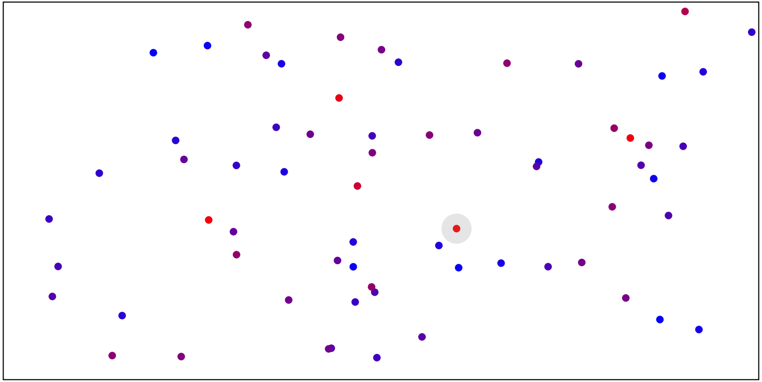
algorithm to remove negative weights [Andersen, Maier 21]

- pick an event with $w_i < 0$
- find nearby events until $\sum_{i\in\mathcal{C}} w_i > 0$
- if $\mathcal{C}$ gets too big (events become resolvable), abort (or add more events)
- else $w_i \to \frac{\sum_{j\in\mathcal{C}} w_j}{\sum_{j\in\mathcal{C}} |w_j|} w_i$

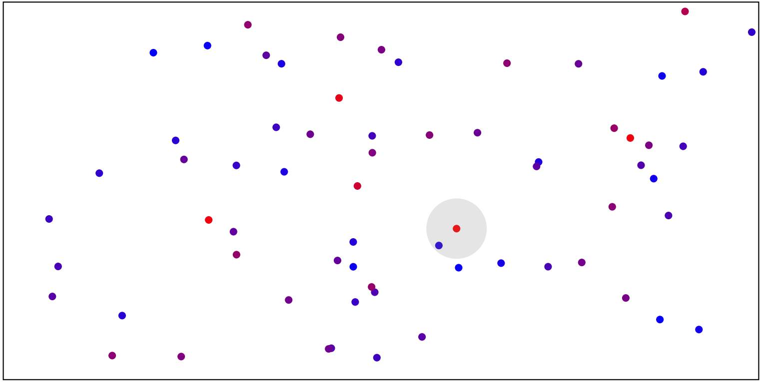
we can remove negative weights without biasing physical observables!

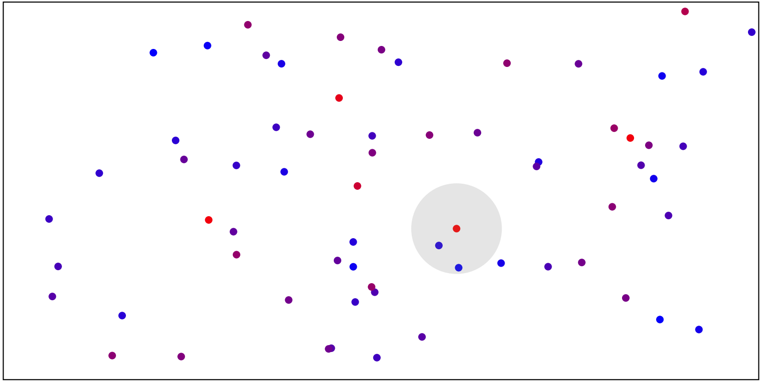
example

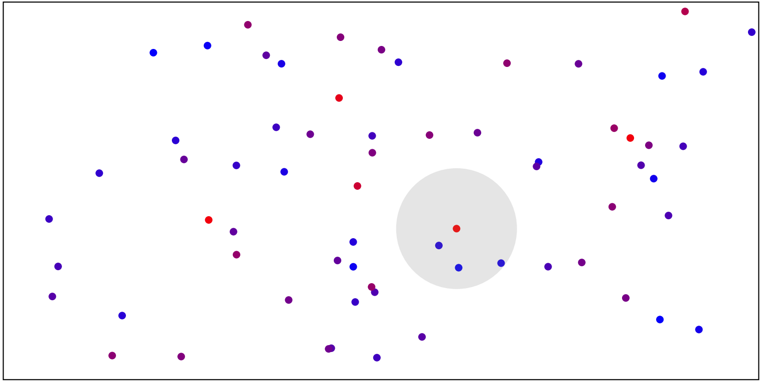


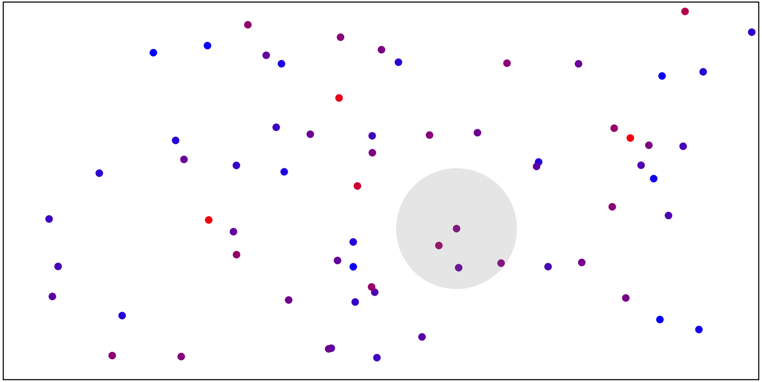


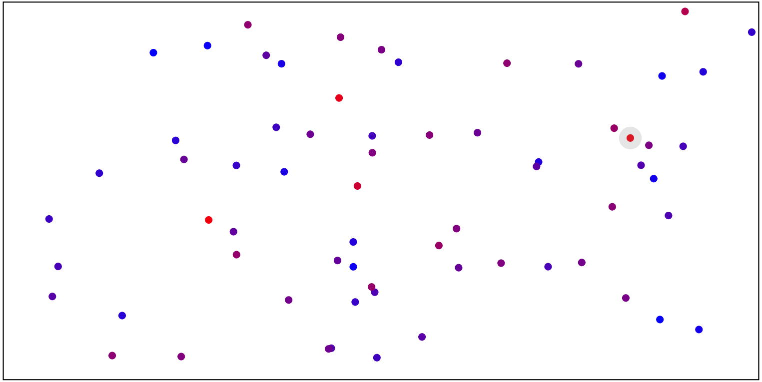
example

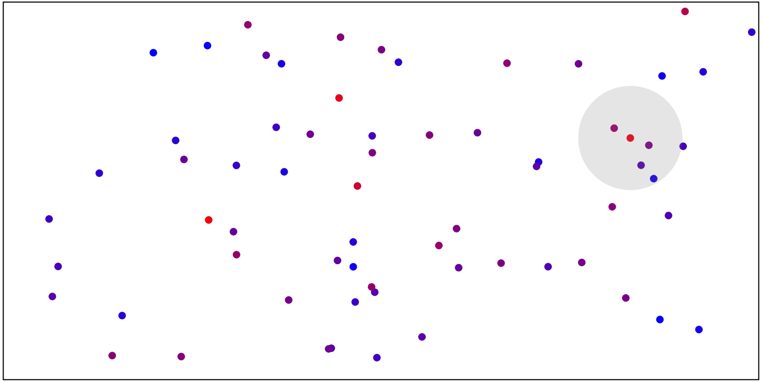


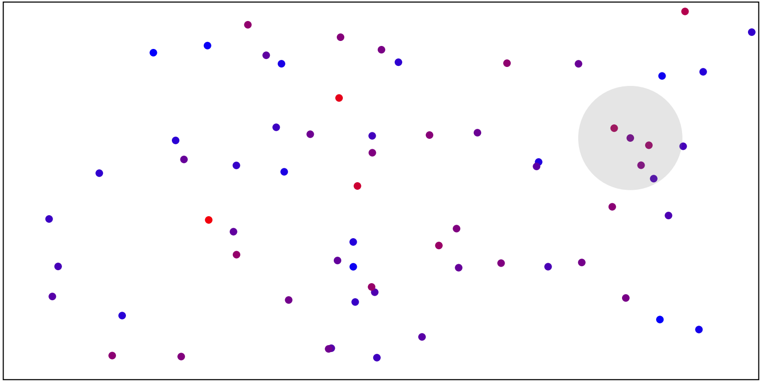


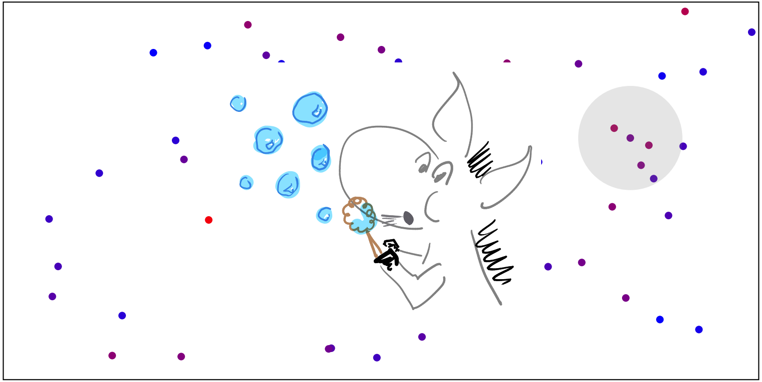






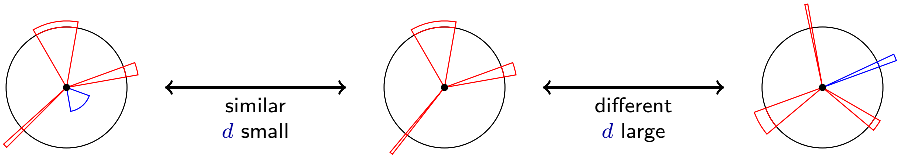






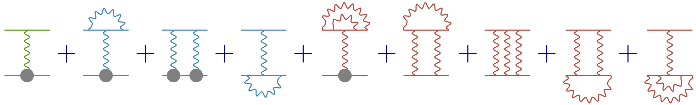
we need to define a metric in event space $d(e_1, e_2) \geq 0$

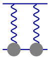
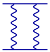
- doesn't really matter how we do this as long as IR safe
  (events with soft photons are near each other)
- ideally: events that look similar are closer to each other than those that don't



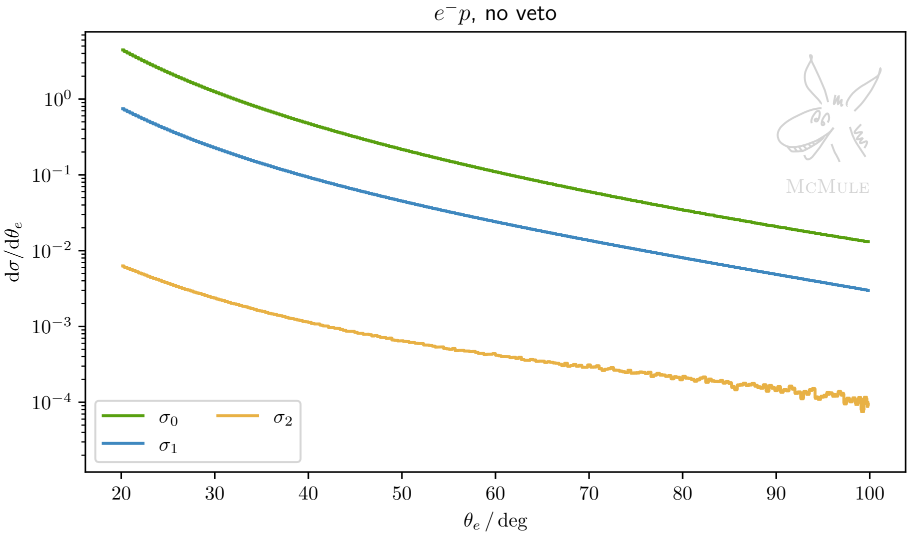
results for MUSE

- $|\vec{p}_{\text{beam}}| = 210\,\text{MeV}$
- cuts: $20^\circ < \theta_\ell < 100^\circ$, $|\vec{p}_\ell| > 15\,\text{MeV}$
- optional ISR veto: no photons with $\sum E_\gamma > 84\,\text{MeV}$ within $100\,\text{mrad}$
- simple dipole model for proton form factor $G_E = \frac{G_M}{1+\kappa} = \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2}$

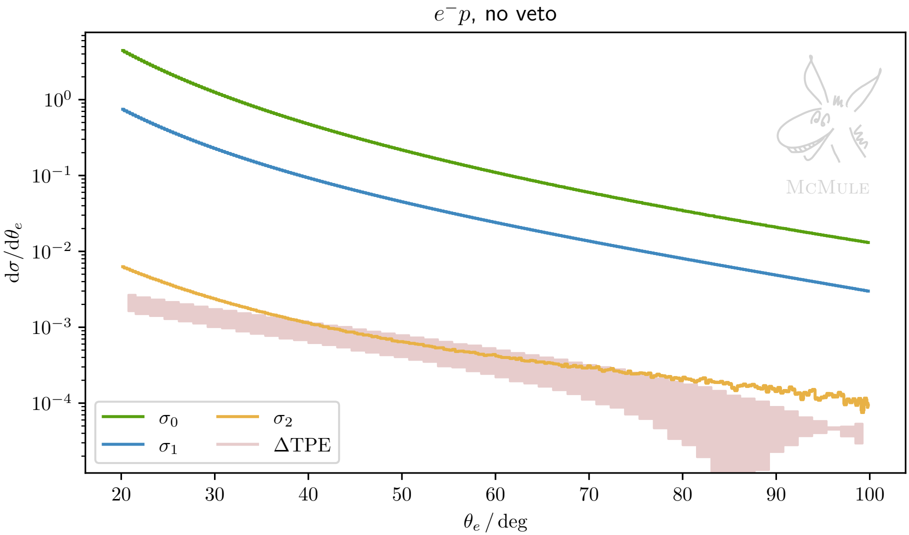
$\sigma \supset$ 

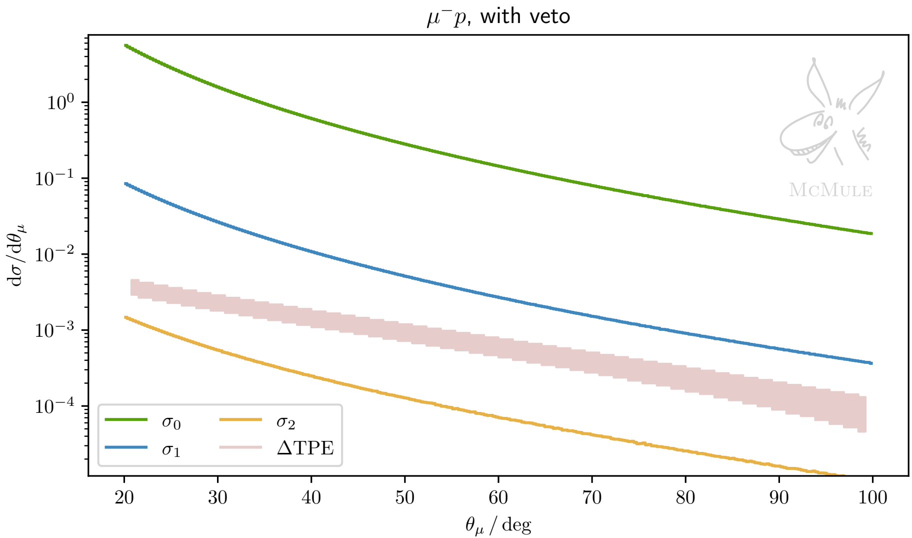
- study TPE effects with $\Delta\text{TPE} =$  $(\Lambda^2) -$  $\Big|_{\Lambda^2=\{0.88,0.71,0.60\}\,\text{GeV}^2}$

all results online `mule-tools.gitlab.io/user-library/l-p-scattering/muse-tpe`
[Engel, Hagelstein, Rocco, Sharkovska, Signer, YU 23]

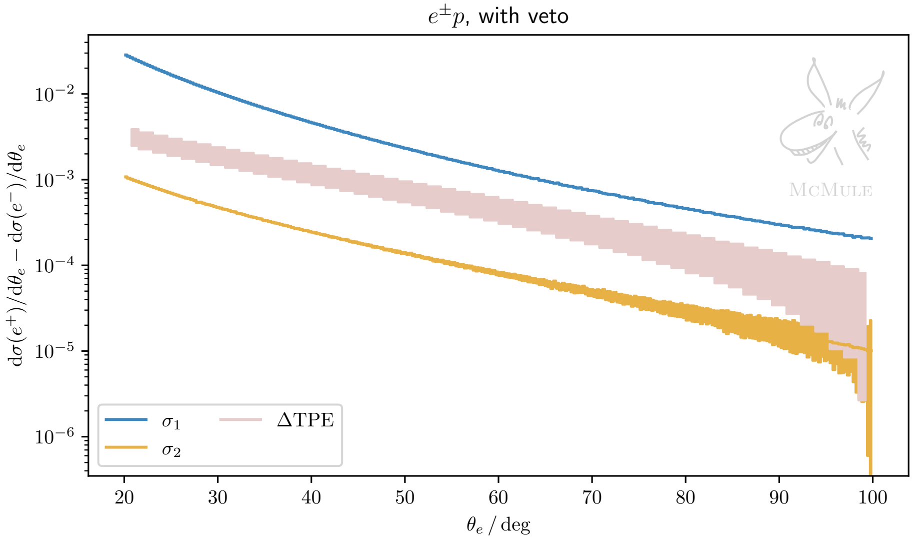


$e^- p$, no veto
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\mathrm{d}\sigma / \mathrm{d}\theta_e$
20
30
40
50
60
70
80
90
100
θ_e / deg
σ_0
σ_1
σ_2
McMule


e^-p, no veto
$d\sigma/d\theta_e$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
20
30
40
50
60
70
80
90
100
θ_e / deg
σ_0
σ_2
σ_1
ΔTPE
McMule



$\mu^- p$, with veto
$d\sigma/d\theta_\mu$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
20 30 40 50 60 70 80 90 100
θ_μ / deg
σ_0
σ_1
σ_2
ΔTPE
McMule


$e^{\pm}p$, with veto
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\mathrm{d}\sigma(e^+)/\mathrm{d}\theta_e - \mathrm{d}\sigma(e^-)/\mathrm{d}\theta_e$
20
30
40
50
60
70
80
90
100
θ_e / deg
σ_1
σ_2
ΔTPE
McMule

point-like calculations

- approximate NNLO for $\ell p \to \ell p\gamma$ from $pp \to 2j + \gamma$
  [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia 23]
- full mass dependence plausible but very difficult

form-factor calculations

- partial N^{3}LO for $\ell p \to \ell p$
- various performance improvements

better hadronic models

- TPE: WIP with Marc Vanderhaeghen
- inelastic: interest from the EIC community?

general

- resummation of soft photon emission

McMule is a state-of-the-art framework for QED

- we have the best QED calculation of pointlike $\ell p \to \ell p$
- it will get better
- we have an okay model of TPE
- it will get better
- we could add inelastic effects for EIC
- please get in touch so we can help you use McMule!
  yannick.ulrich@cern.ch

f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Bern), A.Gurgone (Pavia)
not shown: P.Banerjee (IIT Guwahati), D. Moreno (PSI), D. Radic (PSI)

McMule

mule-tools.gitlab.io