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McMule – a Monte Carlo generator for low energy processes

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why QED?

- extract HVP for $g-2 \Rightarrow \; e^-\mu^\pm \to e^-\mu^\pm$ (MUonE)
  [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 2]
- luminosity measurements $\Rightarrow \; e^+e^- \to e^+e^-$ (Belle, FCC-ee, ...)
  [Banerjee, Engel, Schalch, Signer, YU 21]
- dark sector searches $\Rightarrow \; e^+e^- \to \gamma\gamma$ (PADME, also for luminosity...)
- R ratios $\Rightarrow \; e^+e^- \to$ stuff (DAΦNE, CMD3, ...)
- τ physics $\Rightarrow \; e^+e^- \to \tau^+\tau^-$ (Belle) [Kollatzsch, YU 22]
- proton radius $\Rightarrow \; \ell p \to \ell p$ and $ee \to ee$ (P2, PRad, MUSE)
  [Bucoveanu, Spiesberger 18; Banerjee, Engel, Signer, YU 20; Banerjee, Engel, Schalch, Signer, YU 21; Engel, Hagelstein, Rocco, Sharkovska, Signer, YU 23]
- lepton decays $\Rightarrow \; \ell \to \ell'\nu\bar\nu + \{ee, \gamma, \gamma\gamma\}$ (MEG, Mu3e, Belle, ...)
  [Pruna, Signer, YU 16; YU, 17; Engel, Gnendiger, Signer, YU, 18, Banerjee, Coutinho, Engel, Gurgone, Signer, YU 22]

what is McMule?

fixed-order NNLO QED framework

- provided: matrix elements by us or others
- output: physical cross section for any physical observable
- McMule: phase space generation, subtraction, stabilisation, integration, etc.

- all leptonic $2 \to 2$ processes in QED at NNLO ($+$ a few others)
- integrator & generator
- user defines cuts through arbitrary function that is loaded at run time

McMule

Get the code here: `https://mule-tools.gitlab.io`
Read the docs here: `https://mcmule.readthedocs.io`

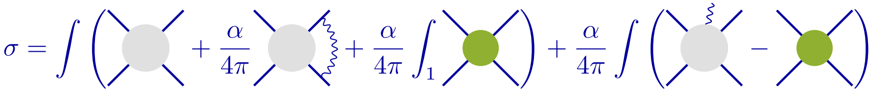
process	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$\ell p \to \ell p$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO
$e^-e^- \to e^-e^-$	Prad 2	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \ell\ell$	VEPP, BES, Daphne, ...	R-ratio	NNLO±
	Belle	τ properties	
$ee \to \gamma\gamma$	Daphne	dark searches	NNLO−
	any e^+e^- collider	luminosity measurement	
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO−
$\mu \to \nu\bar\nu e$	MEG	ALP searches	NNLO+
	DUNE	beam-line profiling	
$\mu \to \nu\bar\nu e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \to \nu\bar\nu eee$	Mu3e	background	NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	+

process	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$\ell p \to \ell p$	P2, Muse, Pr	us and weak charge	NNLO
$e^-e^- \to e^-e^-$	Prad 2	on	NNLO
	MOLLER, ...	low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- col	measurement	NNLO
$ee \to \ell\ell$	VEPP, BES,		NNLO±
	Belle	s	
$ee \to \gamma\gamma$	Daphne	es	NNLO−
	any e^+e^- col	measurement	
$e\nu \to e\nu$	DUNE	θ_W	NNLO−
$\mu \to \nu\bar\nu e$	MEG	es	NNLO+
	DUNE	filing	
$\mu \to \nu\bar\nu e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \to \nu\bar\nu eee$	Mu3e	background	NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	+

goal: world domination

how does McMule work

- written in Fortran 2008, compiled with `meson+ninja`, toolkit in `python3.9`
- links to OpenLoops [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller 19], Collier [Denner, Dittmaier, Hofer 16], handyG [Naterop, Signer, YU 18]
- matrix elements provided as function pointers
- automatic stabilisation for soft emissions at one-loop using the LBK theorem [Engel, Signer, YU 21; Balsach, Bonocore, Kulesza 23]
- automatic IR subtraction using the FKS$^\ell$ scheme [Engel, Signer, YU 19]



- defaults can be overridden if more control is needed
- histogramming can be done in McMule or externally (`LHEF`, `HepMC3`)

"garden hose approach": just dump $\{p_i\}$ and w to file

- limited optimisation with vegas
- plenty of negative weights
- large cancellations happen in histogram
- many more points need to propagate through expensive detector simulation

$\Rightarrow$ prefer early cancellation over late

two observations

1. cross section $\sigma = \int_\mathcal{C} \mathrm{d}\sigma > 0$, irregardless of the size of integration region $\mathcal{C}$
2. experiments have a finite resolution

   (we already knew that because we can't see soft photons)

algorithm to remove negative weights [Andersen, Maier 21]

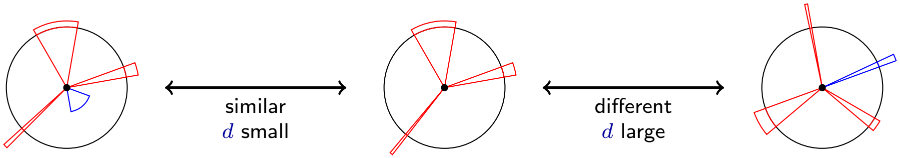
- pick an event with $w_i < 0$
- find nearby events until $\sum_{i\in\mathcal{C}} w_i > 0$
- if $\mathcal{C}$ gets too big (events become resolvable), abort (or add more events)
- else $w_i \to \frac{\sum_{j\in\mathcal{C}} w_j}{\sum_{j\in\mathcal{C}} |w_j|} w_i$

we can remove negative weights without biasing physical observables!

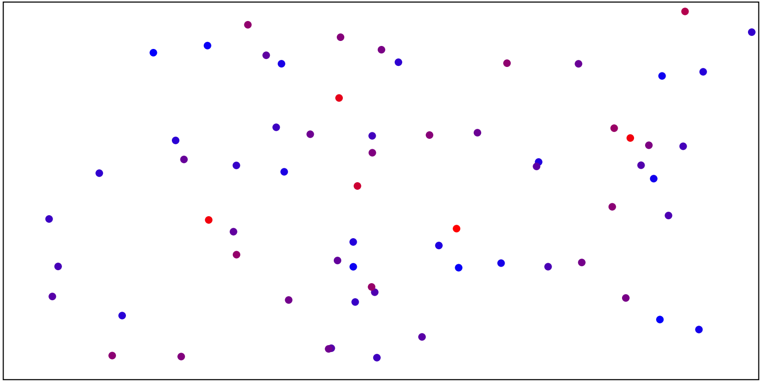
what does nearby mean?

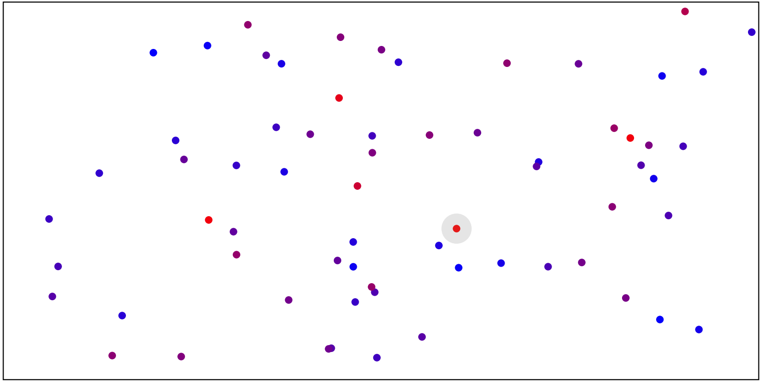
we need to define a metric in event space $d(e_1, e_2) \geq 0$

- doesn't really matter how we do this as long as IR safe
  (events with soft photons are near each other)
- ideally: events that look similar are closer to each other than those that don't
- example for one visible particle $d(e_1, e_2) = \sqrt{\left|\theta_1 - \theta_2\right|^2 + \left|\phi_1 - \phi_2\right|^2}$
- can add ϕ and/or energy information, depending on analysis



example





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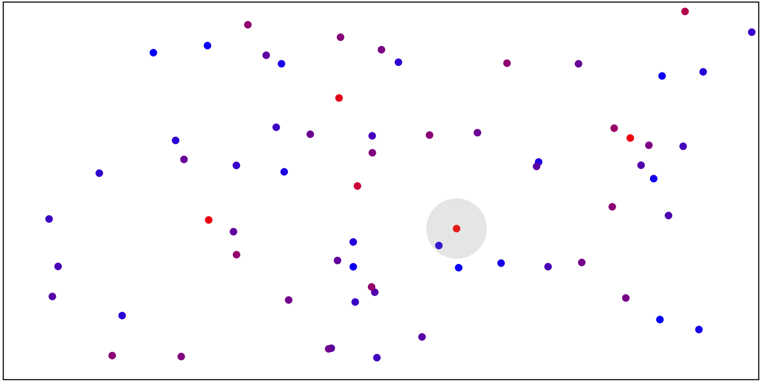
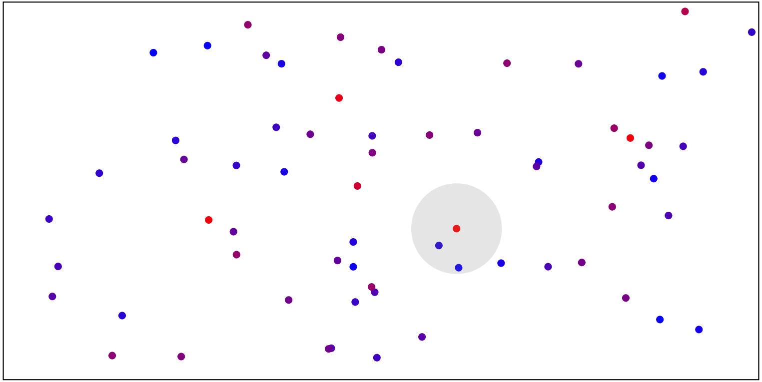
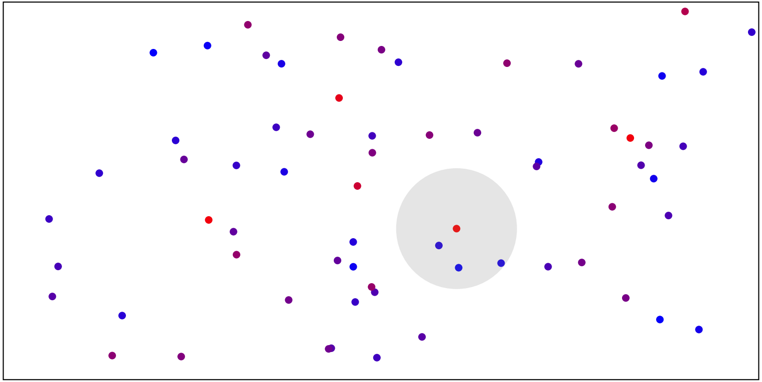


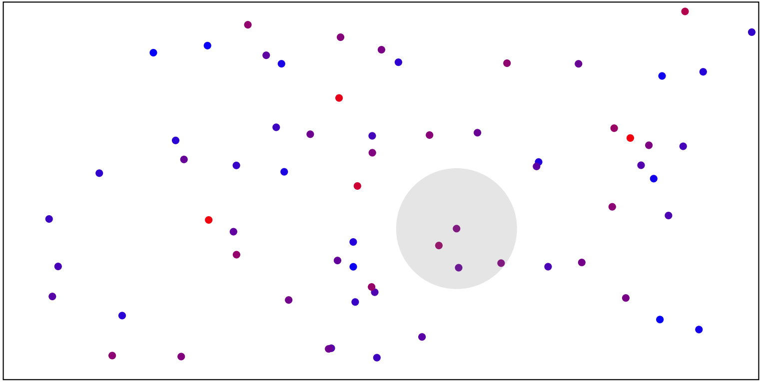
example



example



example



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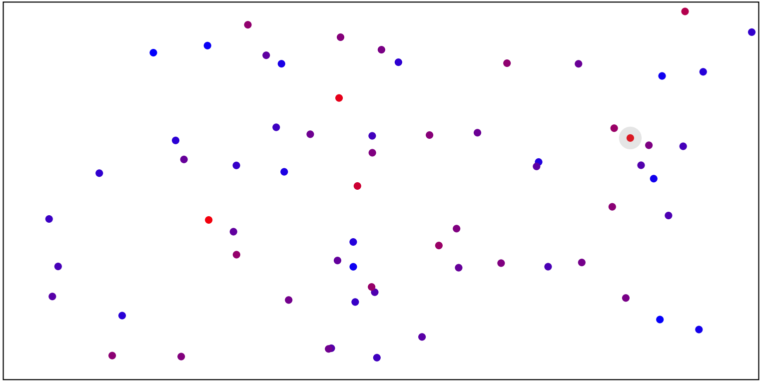

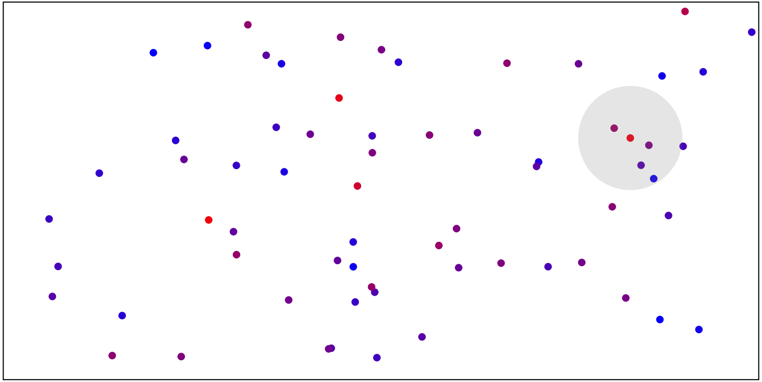

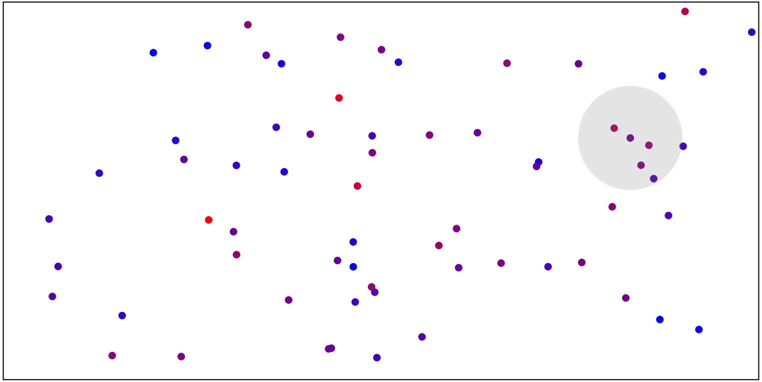

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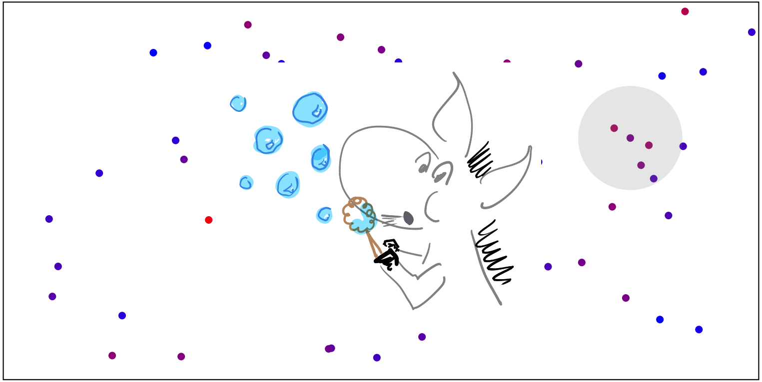
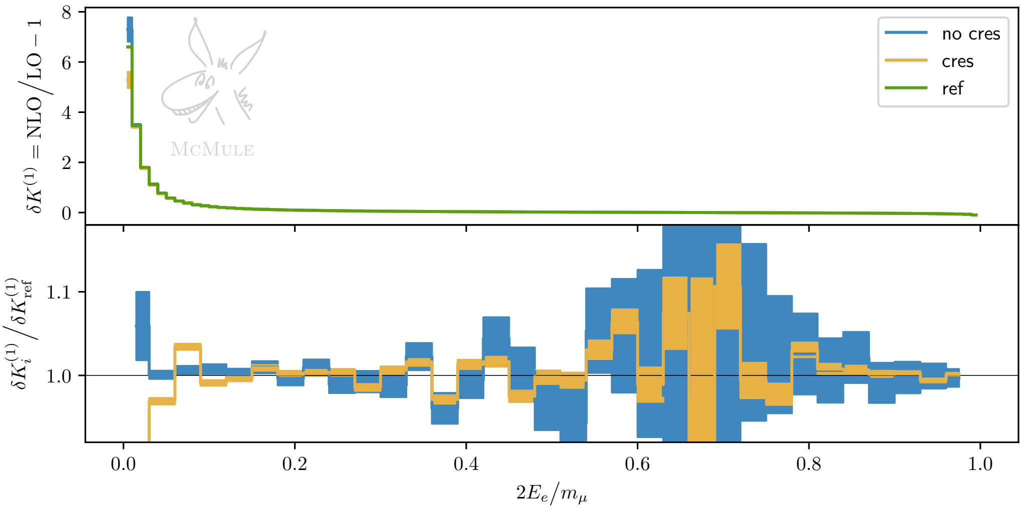
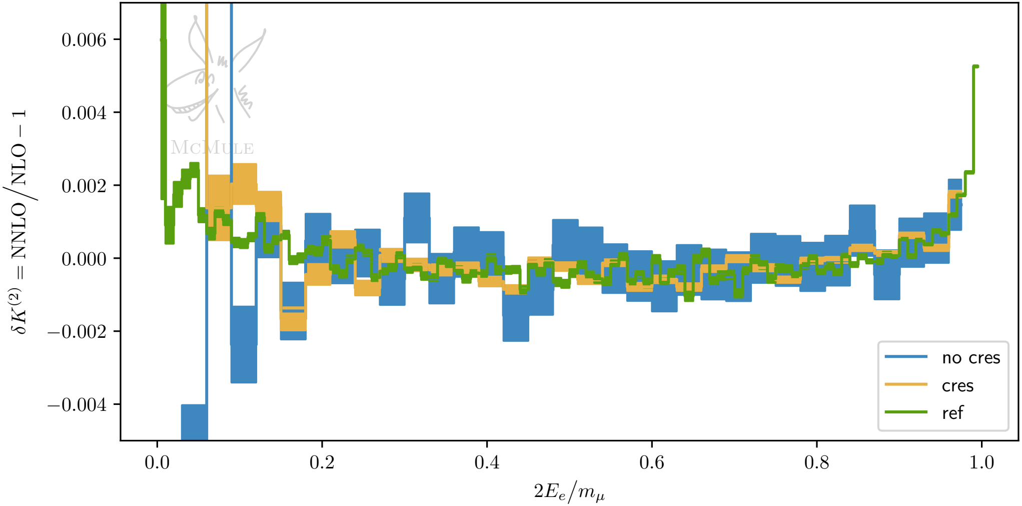




$r \approx 2 \times 10^{-2} \to 2 \times 10^{-5}$, $w_{\min}/\langle w \rangle = -10^5 \to -10^{-3}$


0.006
0.004
0.002
0.000
−0.002
−0.004
$\delta K^{(2)} = \text{NNLO}/\text{NLO} - 1$
0.0
0.2
0.4
0.6
0.8
1.0
$2E_e/m_\mu$
no cres
cres
ref
McMule

iterative cell resampling / subsampling

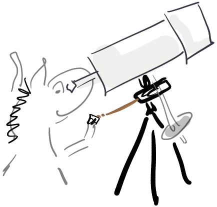
- track an address for each event (eg. importance sampler cell)
- do a dry run of the resampling
- if a $\mathcal{C}$ too big: mark addresses of all $e \in \mathcal{C}$ as bad
- after all events are handled: add more samples in all bad addresses
- actually perform the resampling

subsampling changes the distribution of events but can (if properly configured) remove negative events completely

outlook

more processes

- $e\mu \to e\mu\gamma$ at NNLO with $m_e^2 = m_\mu^2 \ll s$
- $\ell N \to \ell N$ $(N = {}^1H,\ {}^2H,\ {}^{12}C, \dots)$ with improved nuclear models
- $ee \to \pi\pi$, $ee \to \pi\pi\gamma$ at (N)NLO
- form factor contributions $ee \to \gamma^*$ at NNNLO
- inclusion of EW effects



technical improvements

- direct ROOT interface and/or shims to experimental codes
- more automation, especially at NNLO
- YFS shower for soft resummation

f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Durham), A.Gurgone (Pavia)
not shown: P.Banerjee (IIT Guwahati), D. Moreno (PSI), D. Radic (Tubingen)

McMule

`mule-tools.gitlab.io`