

**5<sup>th</sup> MUonE Collaboration Meeting**

**Status of McMULE**  
event generation at NNLO & future

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## fixed-order NNLO QED framework

- provided: matrix elements by us or others
- output: **physical cross section** for any physical observable
- McMULE: phase space generation, subtraction, stabilisation, integration, etc.
- all leptonic  $2 \rightarrow 2$  processes in QED at NNLO (+ a few others)
- stable public version is an integrator
- generator on development branch

Get the code here: <https://mule-tools.gitlab.io>

Read the docs here: <https://mcmule.readthedocs.io>



McMULE

process	experiment	physics motivation	order
$e\mu \rightarrow e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$lp \rightarrow lp$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO
$e^-e^- \rightarrow e^-e^-$	Prad 2	normalisation	NNLO
$e^+e^- \rightarrow e^+e^-$	MOLLER, ...	$\sin^2 \theta_W$ at low $Q^2$	
$ee \rightarrow ll$	any $e^+e^-$ collider	luminosity measurement	NNLO
$ee \rightarrow \gamma\gamma$	VEPP, BES, Daphne, ...	$R$ -ratio	NNLO±
$ee \rightarrow \nu\bar{\nu}$	Belle	$\tau$ properties	
$e\nu \rightarrow e\nu$	DUNE	dark searches	NNLO-
$\mu \rightarrow \nu\bar{\nu}e$	DUNE	luminosity measurement	
$\mu \rightarrow \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	flux & $\sin^2 \theta_W$	NNLO-
$\mu \rightarrow \nu\bar{\nu}eee$	MEG	ALP searches	NNLO+
$ee \rightarrow \pi\pi$	DUNE	beam-line profiling	
$ee \rightarrow ll\gamma$	VEPP, BES, Daphne, ...	background	NLO
$ee \rightarrow ll\gamma$	VEPP, BES, Daphne, ...	background	NLO
$ee \rightarrow \pi\pi$	VEPP, BES, Daphne, ...	$R$ -ratio	+
$ee \rightarrow ll\gamma$	VEPP, BES, Daphne, ...	$R$ -ratio	+

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$e^-e^- \rightarrow e^-e^-$	MOLLER, ...	low $Q^2$	NNLO
$e^+e^- \rightarrow e^+e^-$	any $e^+e^-$ col	measurement	NNLO
$ee \rightarrow ll$	VEPP, BES, Belle		NNLO±
$ee \rightarrow \gamma\gamma$	Daphne	es	NNLO-
$e\nu \rightarrow e\nu$	any $e^+e^-$ col	measurement	NNLO-
$\mu \rightarrow \nu\bar{\nu}e$	DUNE	$\theta_W$	NNLO-
$\mu \rightarrow \nu\bar{\nu}e$	MEG	es	NNLO+
$\mu \rightarrow \nu\bar{\nu}e\gamma$	DUNE	goal: world domination filing	
$\mu \rightarrow \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \rightarrow \nu\bar{\nu}eee$	Mu3e	background	NLO
$ee \rightarrow \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \rightarrow ll\gamma$	VEPP, BES, Daphne, ...	R-ratio	+



## McMULE is predominantly an integrator

- we can calculate  $\sigma = \int d\Phi \mathcal{M} S(\{p_i\})$
- **measurement function**  $S$  can be implemented numerically  $\sim$  cuts, histograms
- event generators produce events (more or less) distributed according to  $w = d\Phi \mathcal{M}$
- trivial solution: dump every event  $\{p_i\}$  and weight  $w$  to file (“garden hose approach”)



minimise  $\{p_i\}$  to propagate through the expensive detector simulation

- the  $w$  can be negative beyond LO and span many orders of magnitude
  - clever sampling can help but not fully solve the problem
  - if  $r \times N$  of  $N$  weights are negative, we need  $\propto 1/(1 - 2r)^2$  events
- $\Rightarrow$  reduce  $r$  as much as possible by cancelling negative weights as early as possible

... at NLO for simplicity

$$\sigma_{\text{NLO}} = \int \text{[tree]} + \frac{\alpha}{4\pi} \int \text{[1-loop]} + \frac{\alpha}{4\pi} \int \text{[2-loop]}$$

- slicing: fairly few negative weights **but** numerically construct  $\log \omega_c$

$$= \int \underbrace{\left( \text{[tree]} + \frac{\alpha}{4\pi} \text{[1-loop]} + \frac{\alpha}{4\pi} \int_1 \text{[2-loop]} \right)}_{\text{mostly } > 0} + \frac{\alpha}{4\pi} \int_{\omega > \omega_c} \underbrace{\text{[2-loop]}}_{> 0}$$

- subtraction: easier integration **but** lots and lots of negative weights ( $\mathcal{O}(5\%)$  at NLO, more at NNLO)

$$= \int \underbrace{\left( \text{[tree]} + \frac{\alpha}{4\pi} \text{[1-loop]} + \frac{\alpha}{4\pi} \int_1 \text{[2-loop]} \right)}_{\text{mostly } > 0} + \frac{\alpha}{4\pi} \int \underbrace{\left( \text{[2-loop]} - \text{[2-loop]} \right)}_{\text{whatever}}$$

## two observations

- ① cross section  $\sigma = \int_{\mathcal{C}} d\sigma > 0$ , irregardless of the size of integration region  $\mathcal{C}$
- ② experiments have a finite resolution  
(we already knew that because we can't see soft photons)

## algorithm to remove negative weights [Andersen, Maier 21]

- pick an event with  $w_i < 0$
- find nearby events until  $\sum_{i \in \mathcal{C}} w_i > 0$
- if  $\mathcal{C}$  gets too big (events become resolvable), abort (or add more events)
- else  $w_i \rightarrow \frac{\sum_{j \in \mathcal{C}} w_j}{\sum_{j \in \mathcal{C}} |w_j|} w_i$

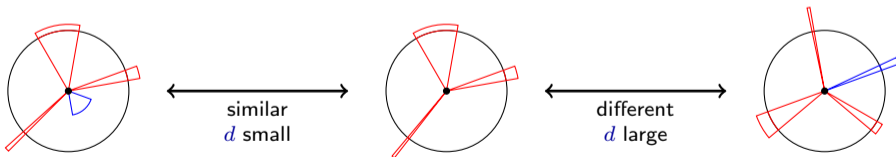
we can remove negative weights without biasing physical observables!

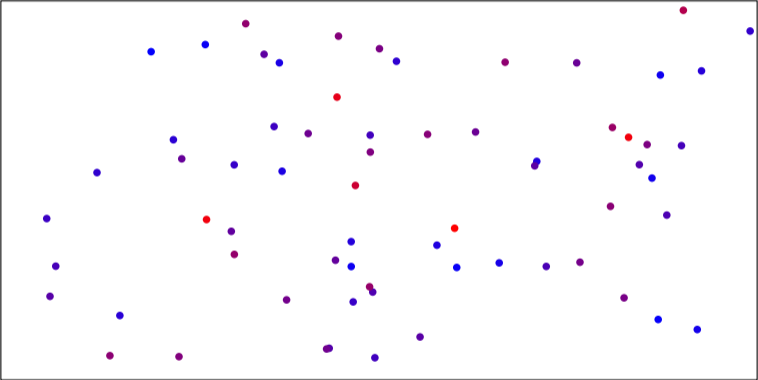
we need to define a metric in event space  $d(e_1, e_2) \geq 0$

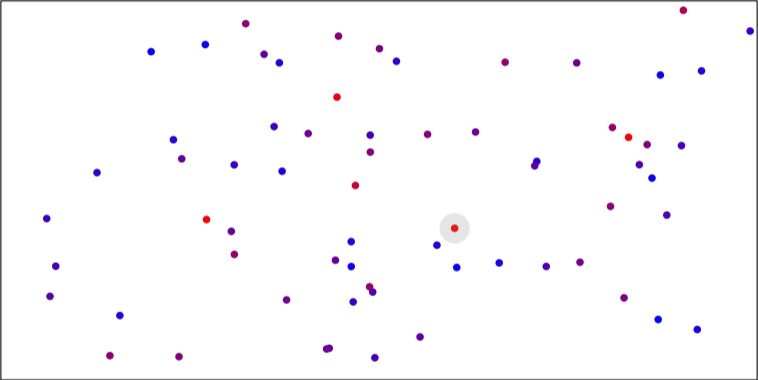
- doesn't really matter how we do this as long as IR safe (events with soft photons are near each other)
- ideally: events that look similar are closer to each other than those that don't

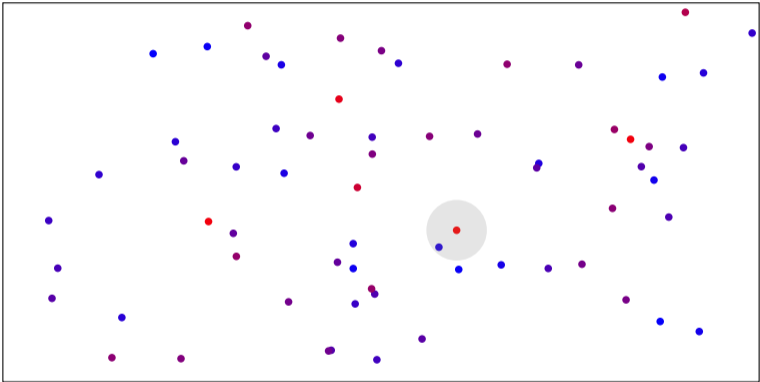
- MUonE example: 
$$d(e_1, e_2) = \sqrt{|\theta_1^e - \theta_2^e|^2 + |\theta_1^\mu - \theta_2^\mu|^2}$$

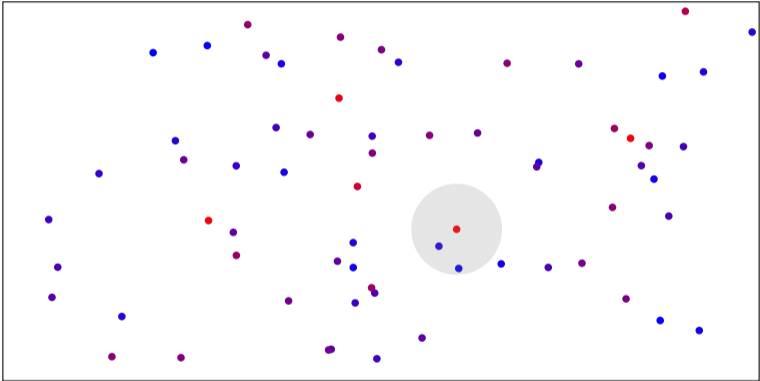
- can add  $\phi$  and/or energy information, depending on analysis

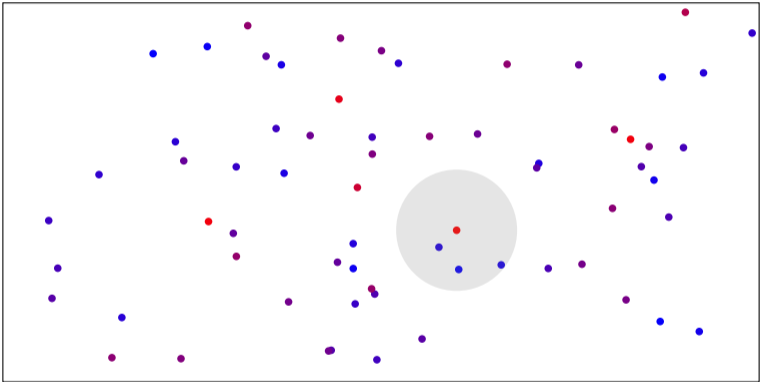


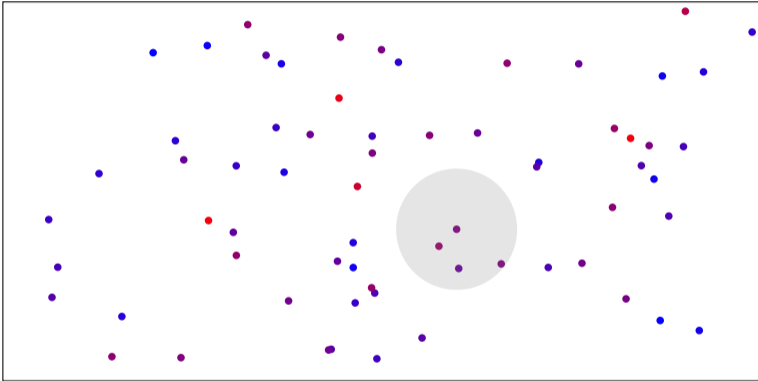


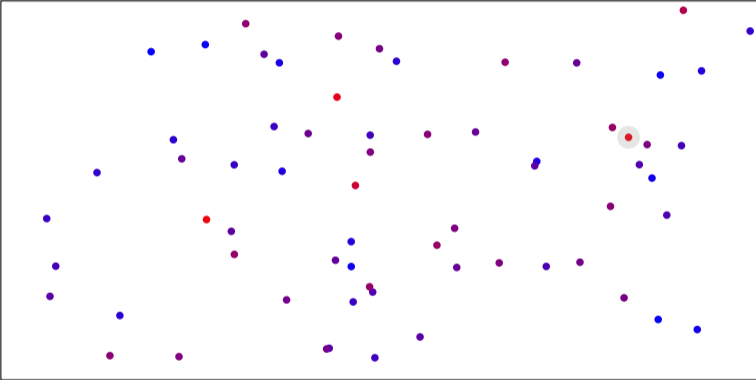


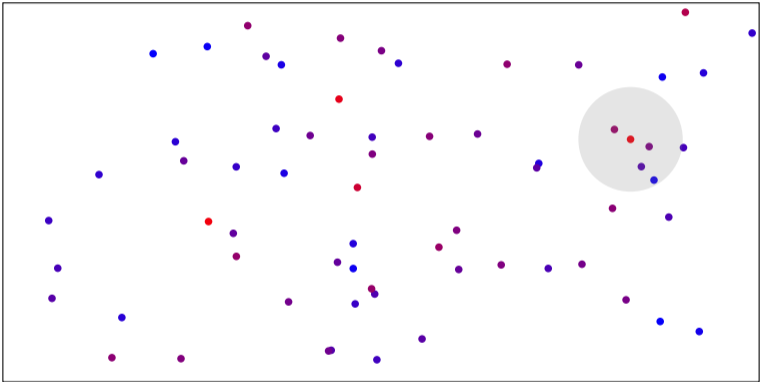


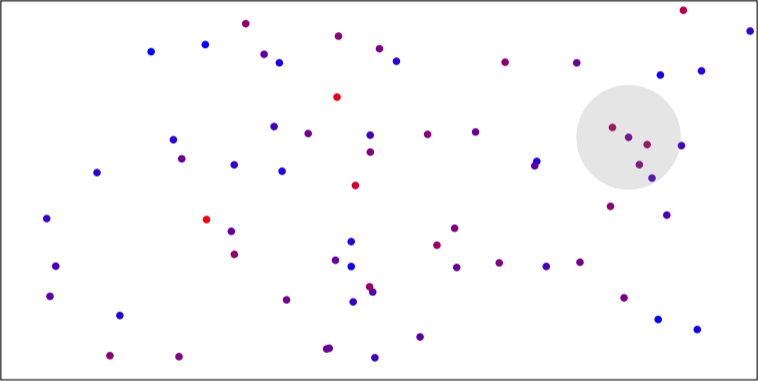


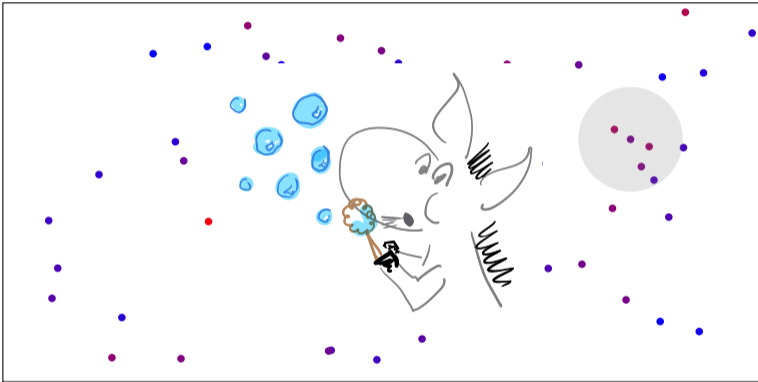


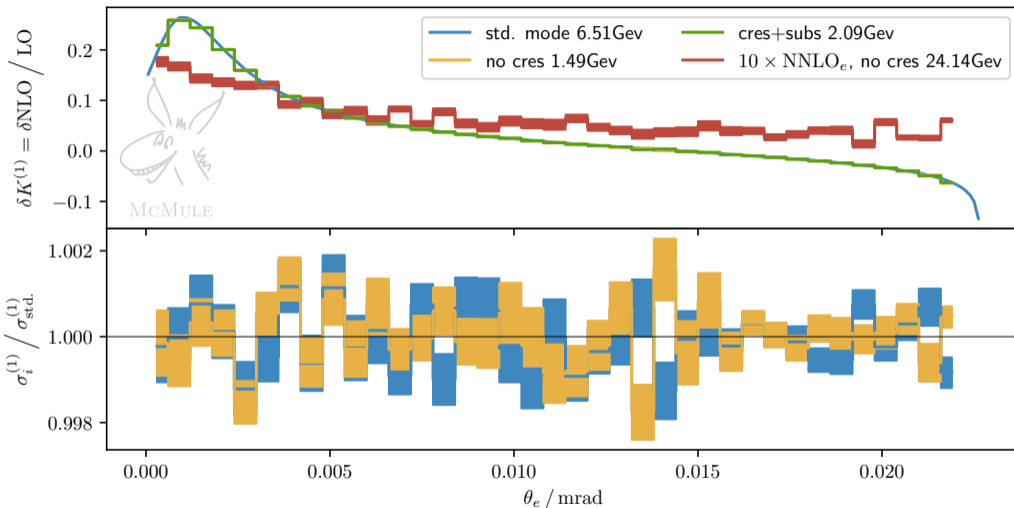












$r \approx 2 \times 10^{-2} \rightarrow 2 \times 10^{-5}$ ,  $w_{\text{min}} / \langle w \rangle = -10^5 \rightarrow -10^{-3}$ , similar at NNLO

## $e\mu \rightarrow e\mu\gamma$ at NNLO

- adapt QCD calculation of  $pp \rightarrow 2j + \gamma$  [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia 23]
- no conceptual problems if photon is not collinear
- **timeline:** this year (or next)

## $N^3\text{LO}$ calculation from $ee \rightarrow \gamma^* \Rightarrow$ Matteo's talk

- implemented two-loop  $ee \rightarrow \gamma^*\gamma$  with  $m_e = 0$  [Badger, Kryś, Moodie, Zoia 23]
- use this with three-loop  $ee \rightarrow \gamma^*$  [Fael, Lange, Schönwald, Steinhauser 22]
- soft emissions understood [Engel 23]
- major problem: hard-collinear emission
- calculation with  $m_e \neq 0$  extremely challenging..
- **timeline:** few years

also:  $l^{12}C \rightarrow l^{12}C$  (in connection  $lp \rightarrow lp$ )

### $e\mu \rightarrow e\mu$ at N<sup>3</sup>LO

- people are working on  $e\mu \rightarrow e\mu$  at three-loop with  $m_e = 0$  (any maybe  $m_\mu = 0$ )
- massification to go to  $m_e > 0$  is understood [Ulrich 23]
- combine  $e\mu \rightarrow e\mu\gamma$  and understanding of  $ee \rightarrow \gamma^*$
- **timeline:** many years

- friendly reminder: NNLO corrections can be as large as  $10^{-3}$
- McMULE is ready to serve MUonE
- we have an NNLO calculation (theoretical error  $\text{few} \times 10^{-5}$ , depending on kinematic region)
- cellular resampling works and (all but) eliminates the negative weights
- N<sup>3</sup>LO is happening, theory error will be below  $10^{-5}$

To whoever does the McMULE-based analysis...

please get in touch so we can help you use McMULE!  
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not shown: P.Banerjee (IIT Guwahati), D. Moreno (PSI), D. Radic (Tubingen)



McMULE

[mule-tools.gitlab.io](https://mule-tools.gitlab.io)