

MPP 2023

McMULE
Monte Carlo for MUons and other LEptons

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fixed-order NNLO QED framework

- provided: matrix elements by us or others
- output: **physical cross section** for any physical observable
- McMULE: phase space generation, subtraction, stabilisation, integration, etc.
- all leptonic $2 \rightarrow 2$ processes in QED at NNLO (+ a few others)
- currently no $ee \rightarrow \pi\pi$ (WIP)!
- stable public version is an integrator (generator WIP)

Get the code here: <https://mule-tools.gitlab.io>



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process	experiment	physics motivation	order
$e\mu \rightarrow e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$lp \rightarrow lp$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO
$e^-e^- \rightarrow e^-e^-$	Prad 2	normalisation	NNLO
$e^+e^- \rightarrow e^+e^-$	MOLLER, ...	$\sin^2 \theta_W$ at low Q^2	
$ee \rightarrow ll$	any e^+e^- collider	luminosity measurement	NNLO
$ee \rightarrow \gamma\gamma$	VEPP, BES, Daphne, ...	R -ratio	NNLO±
$ee \rightarrow \nu\bar{\nu}$	Belle	τ properties	
$e\nu \rightarrow e\nu$	Daphne	dark searches	NNLO-
$\mu \rightarrow \nu\bar{\nu}e$	any e^+e^- collider	luminosity measurement	
$\mu \rightarrow \nu\bar{\nu}e\gamma$	DUNE	flux & $\sin^2 \theta_W$	NNLO-
$\mu \rightarrow \nu\bar{\nu}ee$	MEG	ALP searches	NNLO+
$ee \rightarrow \pi\pi$	DUNE	beam-line profiling	
$ee \rightarrow \ell\ell\gamma$	MEG, Mu3e, Pioneer	background	NLO
$ee \rightarrow \ell\ell\gamma$	MEG, Mu3e	background	NLO
$ee \rightarrow \pi\pi$	VEPP, BES, Daphne, ...	R -ratio	+
$ee \rightarrow \ell\ell\gamma$	VEPP, BES, Daphne, ...	R -ratio	+

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$lp \rightarrow lp$	P2, Muse, Prad 2	us and weak charge	NNLO
$e^-e^- \rightarrow e^-e^-$	MOLLER, ...	low Q^2	NNLO
$e^+e^- \rightarrow e^+e^-$	any e^+e^- col	measurement	NNLO
$ee \rightarrow ll$	VEPP, BES, Belle		NNLO±
$ee \rightarrow \gamma\gamma$	Daphne	es	NNLO-
$e\nu \rightarrow e\nu$	any e^+e^- col	measurement	NNLO-
$\mu \rightarrow \nu\bar{\nu}e$	DUNE	θ_W	NNLO-
	MEG	es	NNLO+
	DUNE	goal: world domination filing	
$\mu \rightarrow \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \rightarrow \nu\bar{\nu}eee$	MEG, Mu3e	background	NLO
$ee \rightarrow \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \rightarrow ll\gamma$	VEPP, BES, Daphne, ...	R-ratio	+



steal from QCD where ever possible

	problem	solution	what?	error
①	lots of masses	massification	expand in m_e^2/Q^2	$10^{-3}\alpha^2$ v. full
②	numerical issues in real corrections	NTS stabilisation	expand in $E_\gamma/\sqrt{Q^2}$	$10^{-2}\alpha^2$ v. quad
③		jettification	expand in $\cos\theta \rightarrow 1$	$10^{-2}\alpha^3$ (exp.)
	phase space	FKS ^ℓ	YFS-inspired subtraction scheme	0

- NNLO double-boxes: ①
- NNLO real-virtual: ②
- N³LO real-virtual-virtual: ①, ②, ③

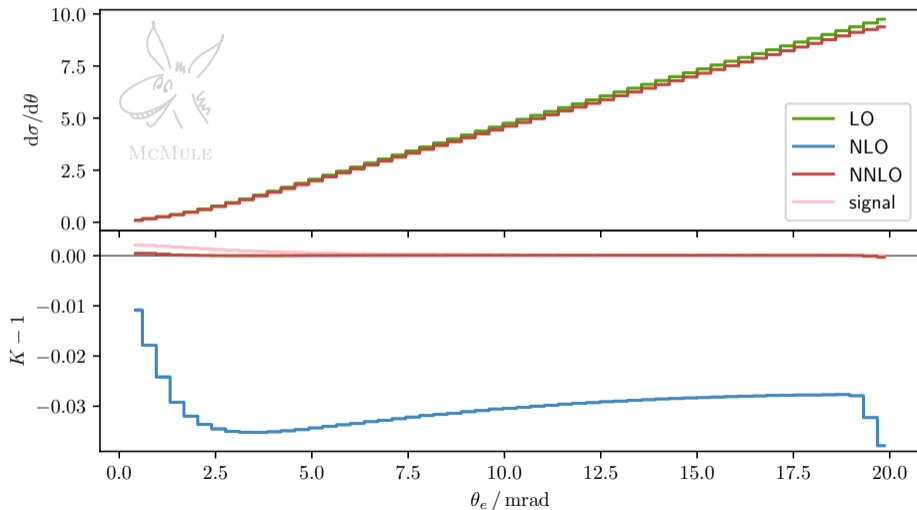


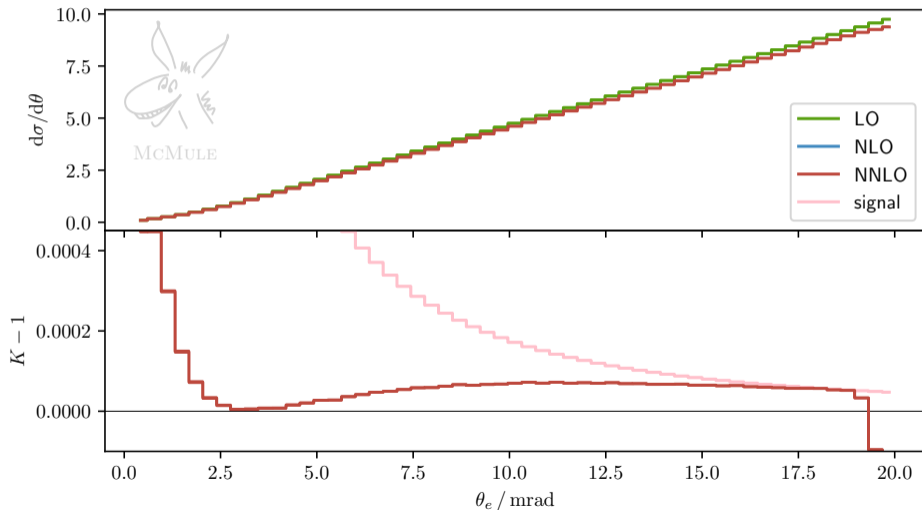
- motivation: see afternoon & tomorrow
- first partial NNLO calculations [McMule 20], [Mesmer 20]
- full NNLO calculation with massification [McMule+Padua++ 22]
- run for 2.5 CPU yr
(290 kWh energy / 1300 kettles / 3.5 kgCO₂e)
- $E_{\text{beam}} = 1 \text{ GeV}$, $E_e > 1 \text{ GeV}$, $\theta_\mu > 0.3 \text{ mrad}$, $0.9 < \theta_\mu / \theta_\mu^{\text{el}} < 1.1$

all results and data:

`mule-tools.gitlab.io/user-library/mu-e-scattering/muone-full-legacy`



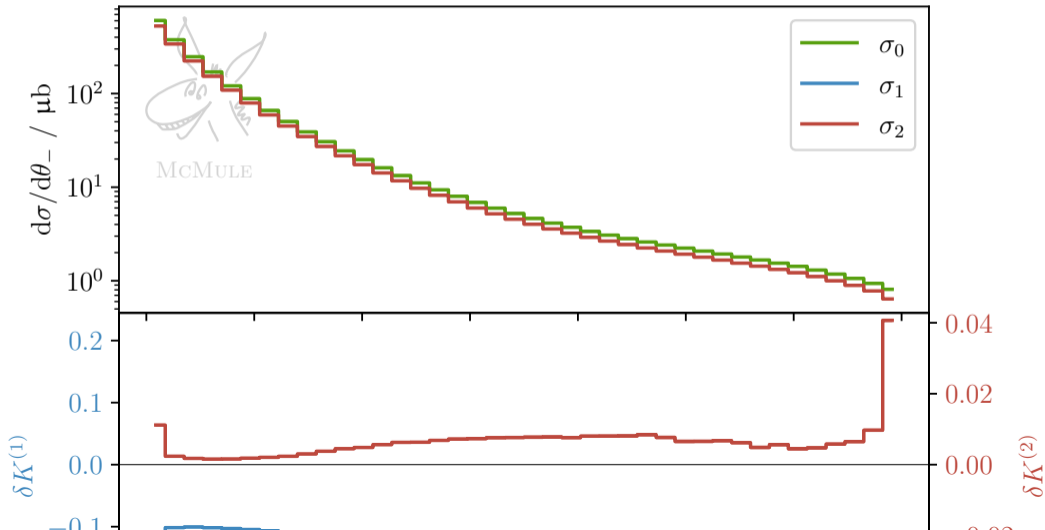




- motivation: luminosity measurement
- full NNLO photonic (i.e. no VP) w/ massification [McMule 21]
- VP effects in Babayaga and elsewhere
- run for 0.8 CPU yr
- $\sqrt{s} = 1020 \text{ MeV}$, $20^\circ < \theta_{\pm} < 160^\circ$, $|180^\circ - \theta_+ - \theta_-| < \zeta_{\text{max}} = 10^\circ$

all results and data:

`mule-tools.gitlab.io/user-library/bhabha-scattering`



- you guys seem to want a generator. we've listened [YU 2?]
- subtraction makes this tricky, rely on cell resampling [Andersen, Maier 22]
- eliminate negative weights without bias by defining distance in event space, eg.

$$d(\theta_{e,i}, \theta_{\mu,i}; \theta_{e,j}, \theta_{\mu,j}) = \sqrt{|\theta_{e,i} - \theta_{e,j}|^2 + |\theta_{\mu,i} - \theta_{\mu,j}|^2}$$



- finalise event generator
- YFS resummation
- $ee \rightarrow \pi\pi$: electronic NNLO [McMule + Bern]
- $ee \rightarrow \gamma\gamma$: pure NNLO-QED (no HLbL) [McMule 2?]
- $ee \rightarrow ee$: w/o massification [McMule + Bonn 2?]
- $ee \rightarrow \mu\mu\gamma$: NNLO with $m_e \sim m_\mu \ll \sqrt{s}$ [McMule + Turin]
- $ee \rightarrow \gamma^*(\rightarrow \mu\mu, \pi\pi, \dots)$: N³ LO with $m_e \ll \sqrt{s}$ [McMule + ⟨ many, many people ⟩]
- anyone's welcome to join, get in touch!





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mule-tools.gitlab.io