DESY Theory Seminar

MUonE

μ-e scattering at 10ppm

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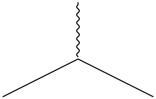
- magnetic moment of a charged lepton: $\vec{\mu} = g\frac{e}{2m}\vec{S}$

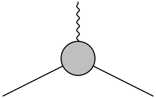
- Dirac: $g_\mu^{\text{Dirac}} = 2$

$$(-ie)\bar{u}\gamma^\mu u =$$

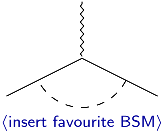


- SM quantum corrections: $g_\mu^{\text{SM}} = 2\times(1+a_\mu) = 2\times(1+F_2(0))$

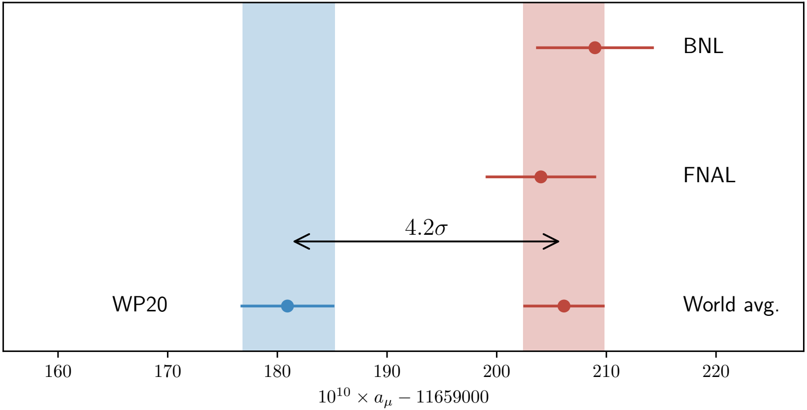
$$(-ie)\bar{u}\Big[F_1(Q^2)\ \gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}Q_\nu}{2m}\Big]u =$$



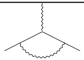
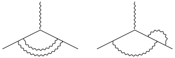
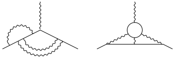
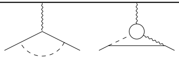
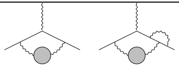
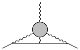
- BSM quantum corrections: $g_\mu^{\text{BSM}} \sim g_\mu^{\text{exp}} - g_\mu^{\text{SM}}$



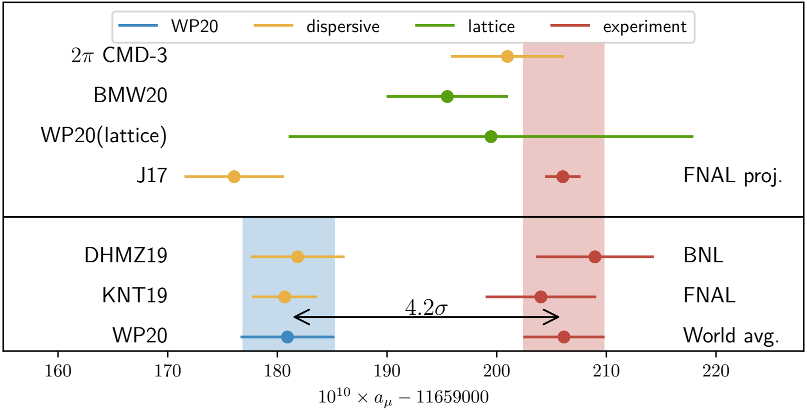
most precise measurement of $g-2$



$\Rightarrow$ needs precise theory

	value	diagrams
QED 1-loop	$\alpha/2\pi = 116\,140\,973$	
QED 2-loop	$-177\,231$	 + 3 others + 1 conspiracy theory
QED 3-loop	$1\,480$	 + 70 others
more QED	-5	
EW	153	
HVP	$6\,845(40)$	 + others
HLbL	$92(17)$	
total	$116\,591\,810(43)$	[$g-2$ white paper 20]
FNAL+BNL	$116\,592\,062(40)$	

largest source of uncertainty & non-perturbative



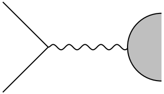
this problem is bigger than $g-2$! [CMD-3 23] [BMW 20]

HVP from dispersive data

using optical theorem $s > 0$

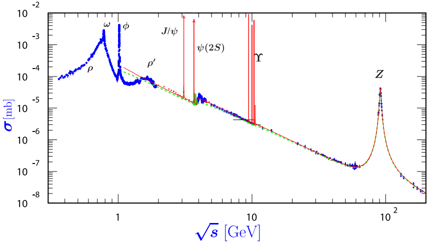
- measure $ee \to$ hadrons
- remove radiative corrections
- extrapolate to $s \to \infty$ using pQCD
- integrate over s

$$a_\mu \supset \int_{4m_\pi^2}^{\infty} \mathrm{d}s \Big(K(s)$$



$$\Big)$$

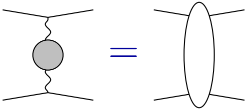
- 72% (78%) of value (uncertainty) from the $ee \to \pi\pi$ channel $s \lesssim 1\,\mathrm{GeV}$

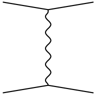


measure low Q^2 regions

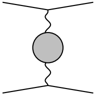
- instead measure in t-channel, i.e. space-like
- no resonances → much cleaner signal
- HVP is loop-induced → much smaller signal $(10^{-3} \times \mathrm{LO})$
- competitive extraction @ 10^{-2}

⇒ goal for MUonE: measure $e\mu \to e\mu$ @ 10^{-5}



$-$  textbook QED

$$a_\mu \supset \int_0^1 \mathrm{d}x \left(K'\left(t = \frac{m_\mu^2 x^2}{x-1}\right) \right)$$



$-$ QED

5+ years,
4+ workshops,
34+ authors

[MUonE 19]

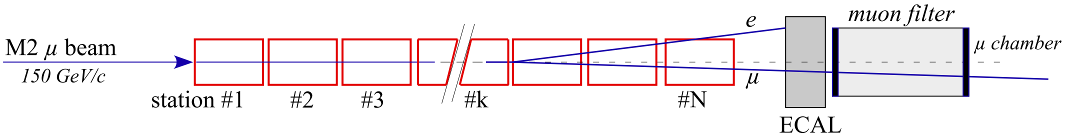
- scattering μ of low-Z material (${}_4\mathrm{Be}$)
- pure t-channel $-s \simeq Q^2 \simeq 0$

$\Rightarrow$ high $s \leftrightarrow$ measure more of the curve

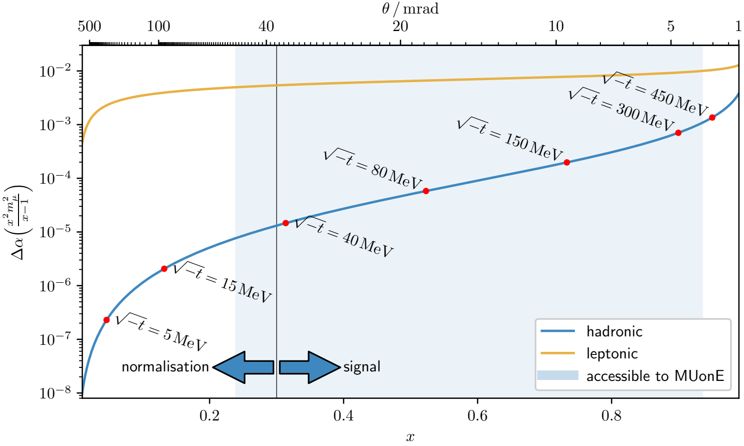
- beam energy needs to be quite high $E_\mu \simeq 160\,\mathrm{GeV}$

$\Rightarrow$ M2 muon beam at CERN North Area

- main measurement: θ_e, θ_μ
  - $+$ E_{beam} for calibration
  - $+$ E_μ for particle ID



cancel systematic effects $\left(\mathrm{d}\sigma/\mathrm{d}\theta\right)_{\mathrm{sig}} \Big/ \left(\mathrm{d}\sigma/\mathrm{d}\theta\right)_{\mathrm{norm}}$



6 MUonE (adjacent) theory workshops over 6+ years

Padua 2017

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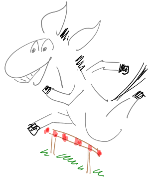
	problem	solution	what?	doable up to?
①	lots of masses	massification	expand in m_e^2/Q^2	LP, three-loop
②	numerical issues in real corrections	NTS stabilisation	expand in $E_\gamma/\sqrt{Q^2}$	NLP, all-orders
③		jettification	expand in $\cos\theta \to 1$	LP, one-loop
	phase space	FKS$^\ell$	YFS-inspired subtraction scheme	all-orders

- NNLO double-boxes: ①
- NNLO real-virtual: ②
- N^{3}LO real-virtual-virtual: ①, ②, ③



	McMule	Mesmer
	mule-tools.gitlab.io	github.com/cm-cc/mesmer
PS	subtraction	slicing
VV	massification	YFS
RV	OpenLoops	hand-tuned Collier
	[Banerjee, Coutinho, Engel, Gurgone, Hagelstein, Kollatzsch, Moreno, Naterop, Proust, Radic, Rocco, Schalch, Signer, Sharkovska, YU]	[Budassi, Carloni Calame, Chiesa, Del Pio, Gurgone, Montagna, Nicrosini, Piccinini, Alacevich, Hasan]

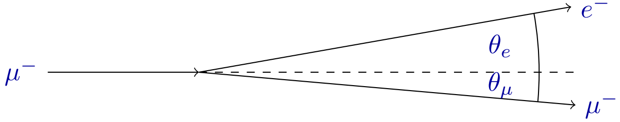
$\Rightarrow$ full agreement

implemented in McMule v0.4.2

side note: new manual, let us know what you think!

https://mule-tools.gitlab.io/manual/

- $\mu^- e^- \to \mu^- e^-$

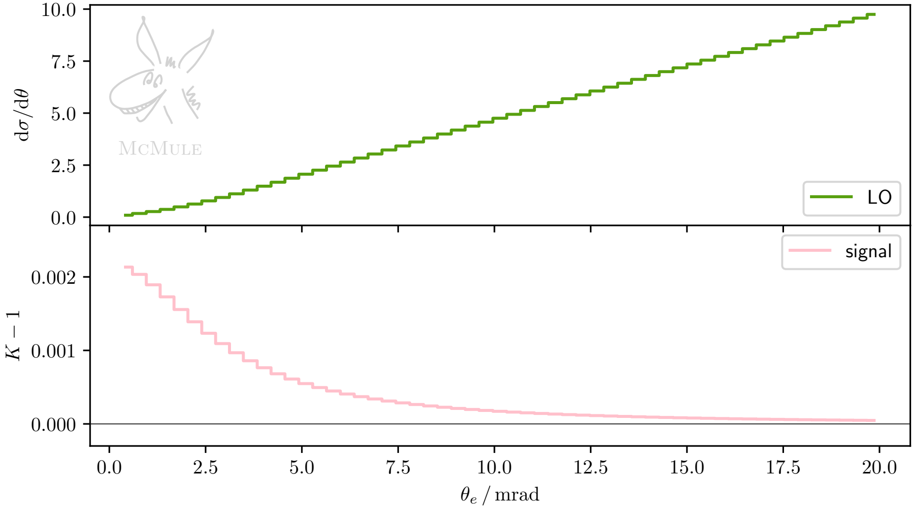


- S1: $E_e > 1\,\mathrm{GeV}$, $\theta_\mu > 0.3\,\mathrm{mrad}$
- run for 2.5 CPU yr
  (290 kWh energy / 3.5 kgCO2e)

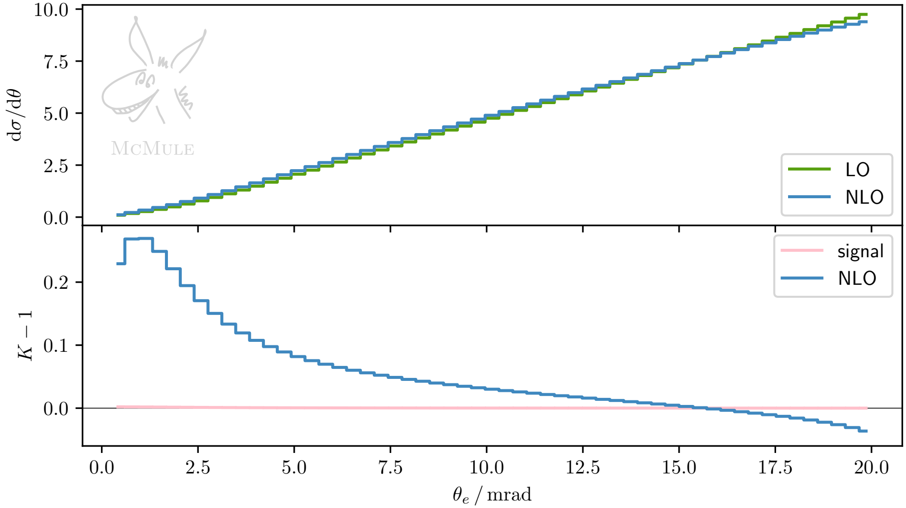
[Broggio, Engel, Ferroglia, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]

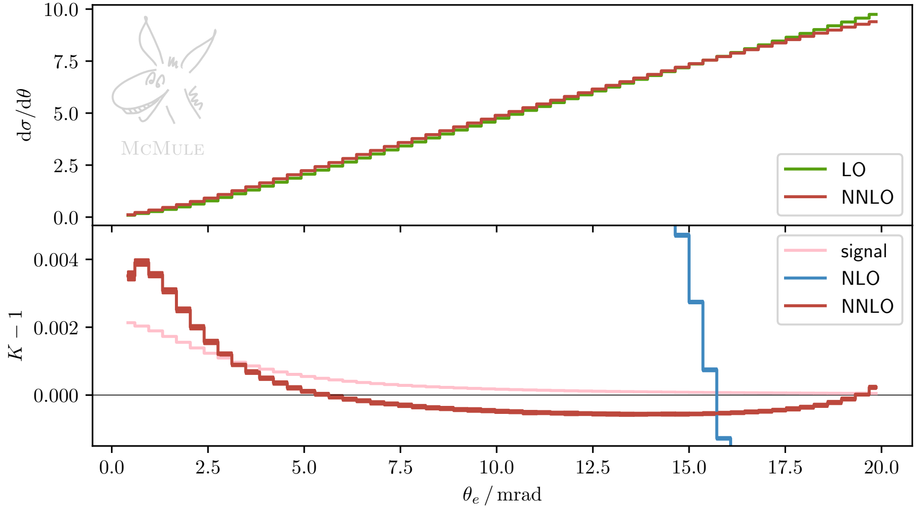
all results and data: https://mule-tools.gitlab.io/user-library/mu-e-scattering/muone-full-legacy/

S1 θ_e spectrum

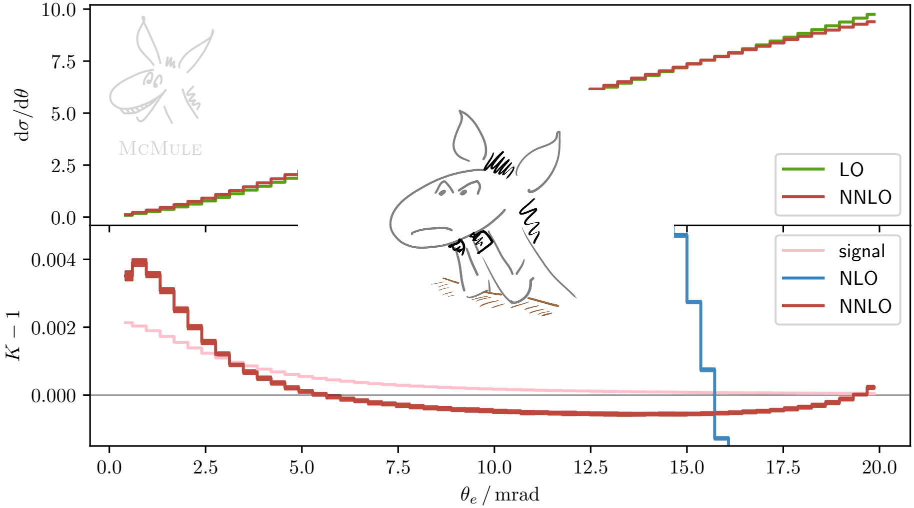


S1 θ_e spectrum



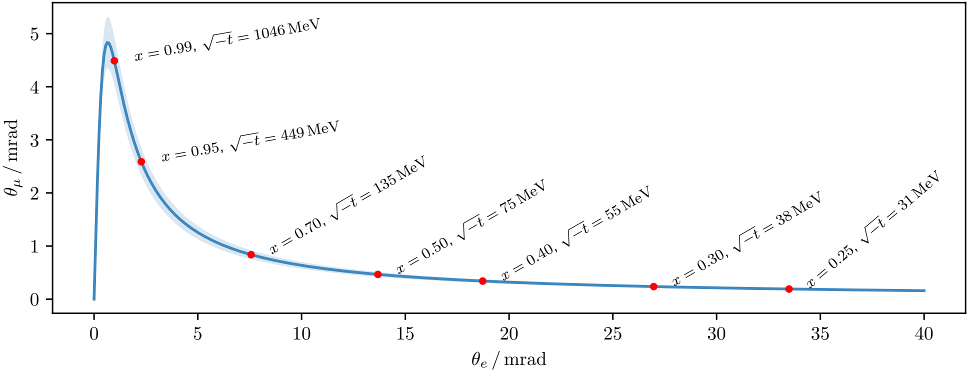

10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
LO
NNLO
0.004
0.002
0.000
K − 1
signal
NLO
NNLO
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θe / mrad

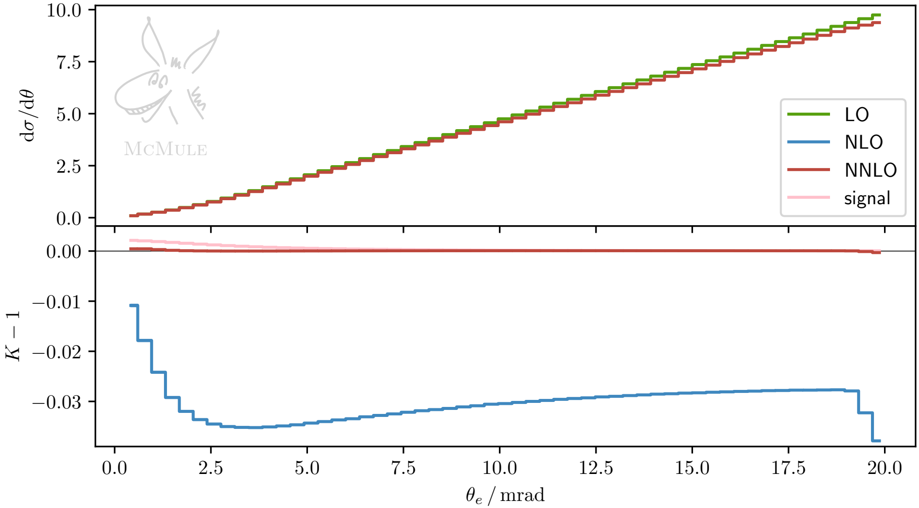
S1 θ_e spectrum



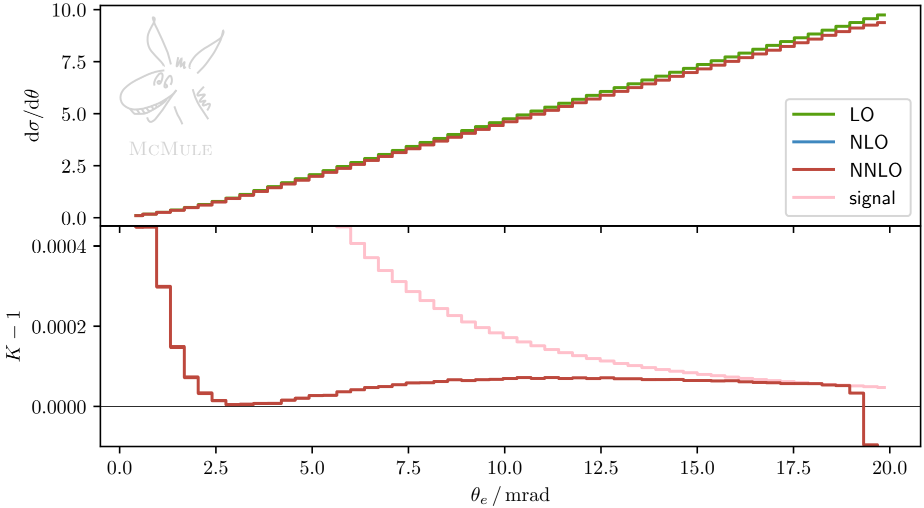
this clearly isn't working

- at this rate ($\sim 10\%$ NLO, $\sim 0.1\%$ NNLO), we would need N^{4}LO to reach 10^{-5}
- most of this is due to hard radition
- S2: same as S1 + needs to be in the band

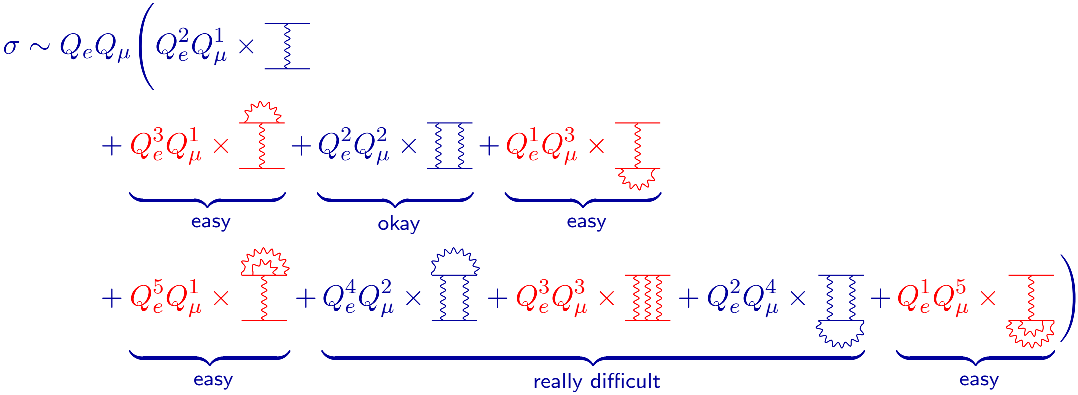



MCMULE
10.0
7.5
5.0
2.5
0.0
$d\sigma/d\theta$
LO
NLO
NNLO
signal
0.00
−0.01
−0.02
−0.03
$K-1$
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad

S2 θ_e spectrum



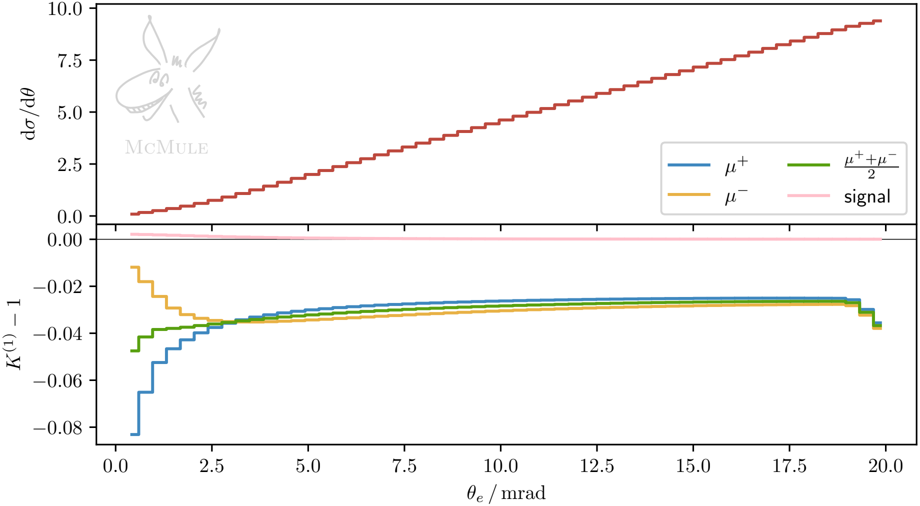
the beam can do both μ^+ and μ^-

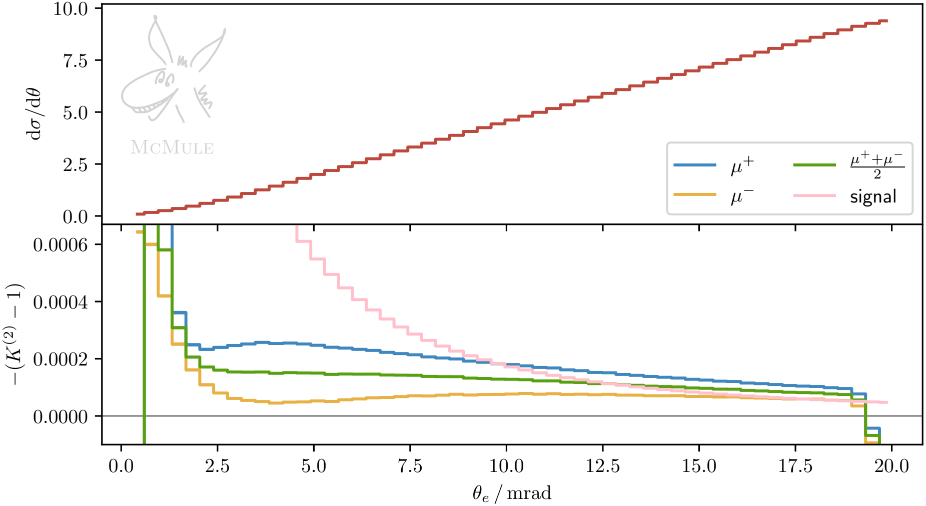


- proposal $\sigma(\mu^+) + \sigma(\mu^-)$

$\Rightarrow$ some of the difficult stuff cancels

playing with the beam

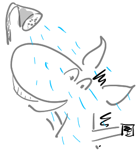




10.0
7.5
5.0
2.5
0.0
$d\sigma/d\theta$
McMule
μ^+
μ^-
$\frac{\mu^+ + \mu^-}{2}$
signal
0.0006
0.0004
0.0002
0.0000
$-(K^{(2)}-1)$
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad

this is obviously missing resummation

- soft: YFS Monte Carlo, up to NNLL (in the works by McMule)
- collinear: QED shower [Carloni Calame 01], up to LL (in the works by Mesmer)

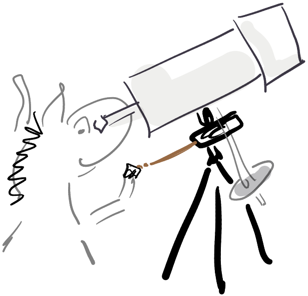


experimentalists need multiple generators

- Mesmer already being used
- McMule forthcoming [YU 2?] using cell resampling [Andersen, Maier 21]

NNLO done, much left to do

- ✓ first NNLO with multiple external masses
  [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]
- ✓ event generation (not in McMule)
- ✓ iterative HVP extraction procedure
  [Fael 18]
- ✓ precision now: $\mathcal{O}(10^{\{-3,-4\}})$, goal: $\mathcal{O}(10^{-5})$
- lots of optimisation still possible
  (observable, beam, polarisation etc)
- resummation (analytic & parton shower)
- partial N^{3}LO ($Q_e^8 Q_\mu^2$)



f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Durham), A.Gurgone (Pavia)
not pictured: P.Banerjee (IIT Guwahati), D.Moreno (PSI), D.Radic (Tubingen)

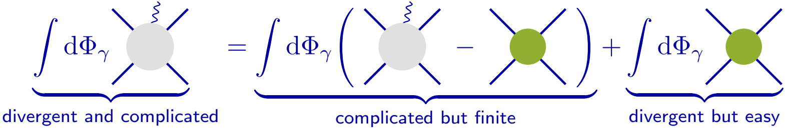
McMule

mule-tools.gitlab.io

- universal soft limit $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f} = \text{finite}$

use this to construct an all-order subtraction scheme FKS$^\ell$[Engel, Signer, YU 19]

- nothing complicated needed higher than $\mathcal{O}(\epsilon^0)$
- only one universal CT: $\hat{\mathcal{E}}$

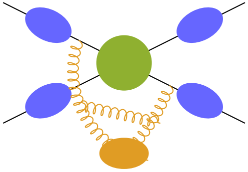


masses are physical in QED $\Rightarrow$ keep masses

- drop polynomially suppressed terms at two-loop $\to$ error $\sim \left(\frac{\alpha}{\pi}\right)^2 \log\frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$
- based on factorisation, SCET, and method of regions

  [Penin 06; Mitov, Moch 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g. $ee \to ee$ at two-loop:

$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$



- soft: process-dependent $S = 1 +$ fermion loops

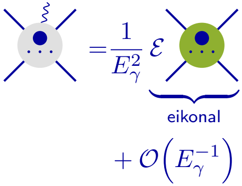
  $\to$ compute separately anyway to combine with hadron loops
- collinear: universal Z, converts $1/\epsilon \to \log(m^2/Q^2)$
- hard: massless calculation

real-virtual (or even real-real-virtual)

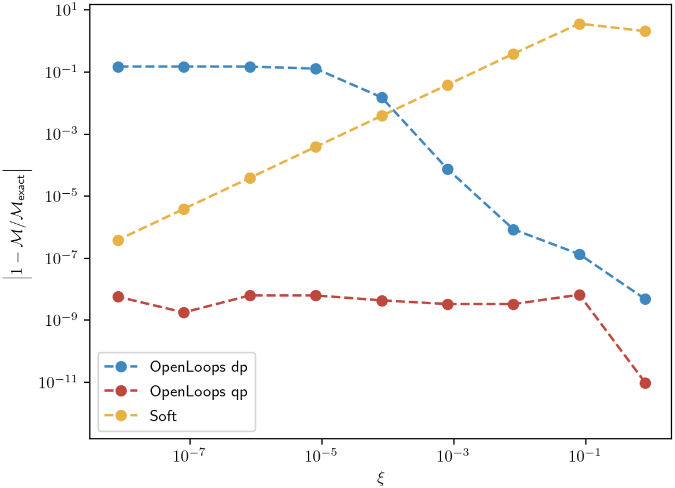
$$\mathcal{M}^{(\ell)}_{n+1} \sim \frac{1}{E_\gamma^2(1-\beta\cos\theta)}$$

- 'trivial' in principle [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini et al. 19]
- extremely delicate numerically for $E_\gamma \to 0$ (or $\cos\theta \to 1$)

$\Rightarrow$ Taylor expand around $E_\gamma = 0$ if small



example $e^+e^- \to e^+e^-\gamma$ @ one-loop



compare with exact calculation in Mathematica

[Banerjee, Engel, Schalch, Signer, YU 21]

real-virtual (or even real-real-virtual)

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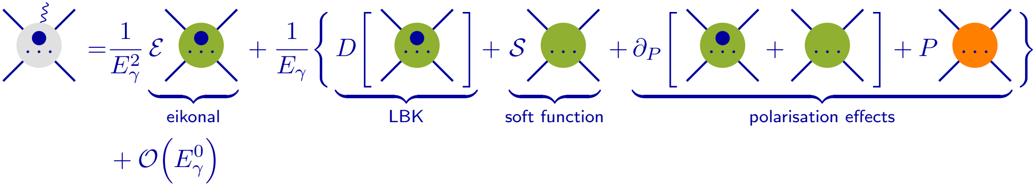
⇒ Taylor expand around $E_\gamma = 0$ if small

- LBK theorem [Low 58; https://inspirehep.net/literature/51370] and extension [Engel, Signer, YU 21; Kollatzsch, YU 22; Engel 23]



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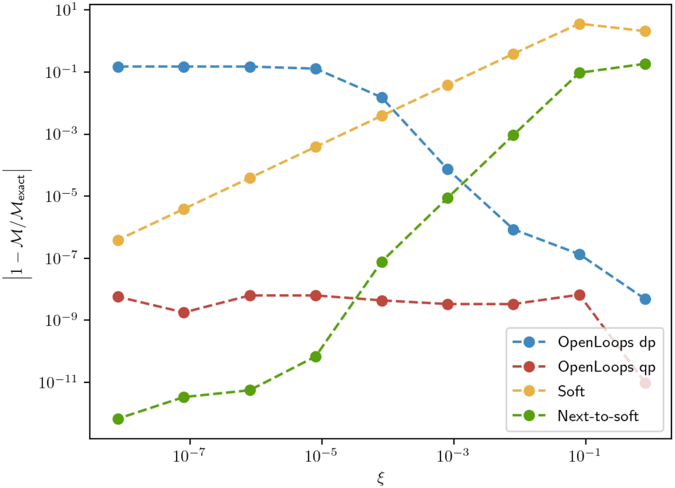

compare with exact calculation in Mathematica

[Banerjee, Engel, Schalch, Signer, YU 21]