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QED at NNLO and beyond for precision experiments

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$g-2$ and the MUonE experiment ($\rightarrow$ Marco's talk)

- luminosity measurements $\Rightarrow$ $e^+e^- \to e^+e^-$ (Belle, FCC-ee, ...)
  [Banerjee, Engel, Schalch, Signer, YU 21]
- dark sector searches $\Rightarrow$ $e^+e^- \to \gamma\gamma$ (PADME, also for luminosity...)
  [Engel, Naterop, Signer, YU, Zoller 2?]
- R ratios $\Rightarrow$ $e^+e^- \to$ stuff (DAΦNE, CMD3, ...)
- τ physics $\Rightarrow$ $e^+e^- \to \tau^+\tau^-$ (Belle) [Kollatzsch, YU 22]
- proton radius $\Rightarrow$ $\ell p \to \ell p$ and $ee \to ee$ (P2, PRad, MUSE)
  [Bucoveanu, Spiesberger 18; Banerjee, Engel, Signer, YU 20; Banerjee, Engel, Schalch, Signer, YU 21]
- lepton decays $\Rightarrow$ $\ell \to \ell' \nu\bar\nu + \{ee, \gamma, \gamma\gamma\}$ (MEG, Mu3e, Belle, ...)
  [Pruna, Signer, YU 16; YU, 17; Engel, Gnendiger, Signer, YU, 18, Banerjee, Coutinho, Engel, Gurgone, Signer, YU 22]

QCD @ LHC	$\Leftrightarrow$	QED @ low & medium energy	
non-abelian	$\gtrsim$	abelian	matrix elements somewhat easier
non-abelian	$\gg$	abelian	IR structure much easier ①
massless fermions	$\ll$	massive fermions	loop amplitudes much harder ②
jets	$<$	exclusive w.r.t.	numerics harder $\supset \log(m^2/Q^2) \equiv L$
		collinear radiation	much harder for small masses ③

stealing from QCD

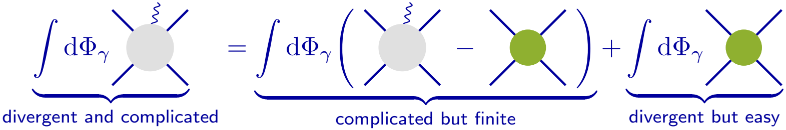
- master integrals (reduction and computation), automated tools, EFT methods
- use dimensional regularisation for IR singularities, not photon mass
- use subtraction method for phase-space integration, not slicing method
- for the future: match fixed-order result to parton shower

soft singularities exponentiate [Yennie, Frautschi, Suura 61]

- universal soft limit $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f} =$ finite

use this to construct an all-order subtraction scheme FKS$^\ell$

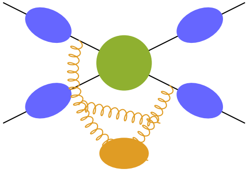
- nothing complicated needed higher than $\mathcal{O}(\epsilon^0)$
- only one universal CT: $\hat{\mathcal{E}}$



masses are physical in QED $\Rightarrow$ keep masses

- drop polynomially suppressed terms at two-loop $\rightarrow$ error $\sim \left(\frac{\alpha}{\pi}\right)^2 \log\frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$
- based on factorisation, SCET, and method of regions
  [Penin 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g. $ee \to ee$ at two-loop:

$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$



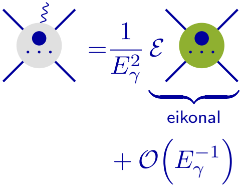
- soft: process-dependent $S = 1 +$ fermion loops
  $\rightarrow$ compute separately anyway to combine with hadron loops
- collinear: universal Z, converts $1/\epsilon \rightarrow \log(m^2/Q^2)$
- hard: massless calculation

real-virtual (or even real-real-virtual)

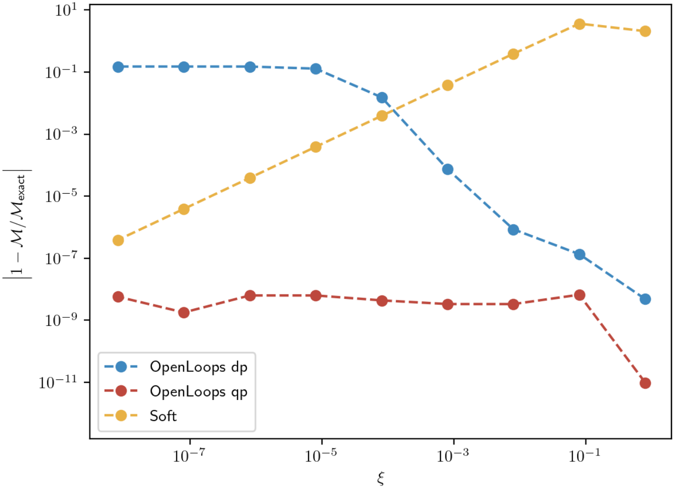
$$\mathcal{M}^{(\ell)}_{n+1} \sim \frac{1}{E_\gamma^2(1-\beta\cos\theta)}$$

- ‘trivial’ in principle [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini et al. 19]
- extremely delicate numerically for $E_\gamma \to 0$ (or $\cos\theta \to 1$)

$\Rightarrow$ Taylor expand around $E_\gamma = 0$ if small



example $e^+e^- \to e^+e^-\gamma$ @ one-loop



compare with exact calculation in Mathematica

[Banerjee, Engel, Schalch, Signer, YU 21]

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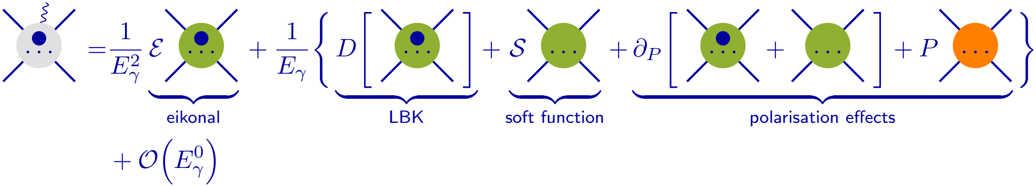
- LBK theorem [Low 58; Burnett, Kroll 67] and extension [Engel, Signer, YU 21; Kollatzsch, YU 22; Engel 23]



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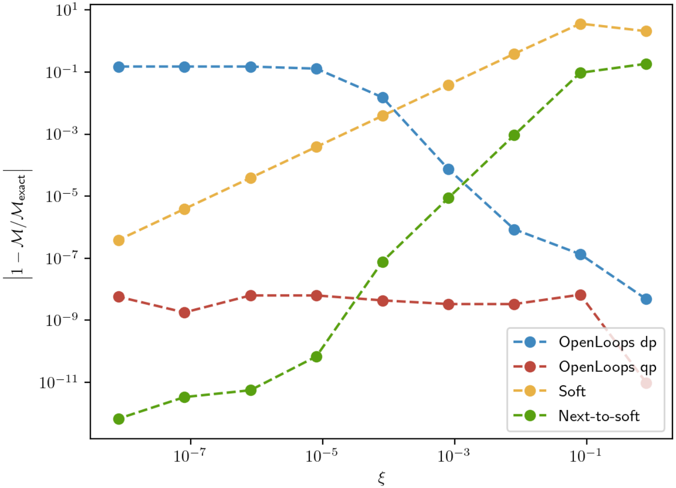

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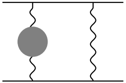
[Banerjee, Engel, Schalch, Signer, YU 21]

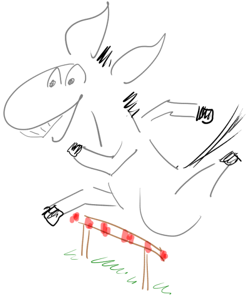
a few more hurdles

- VP diagrams for $e/\mu/\tau/$had$/\dots$ numerically with full mass dependence

 $= \int dQ^2\ \Pi(Q^2) K(Q^2, s, t)$

- collinear pseudo-singularities $\lim_{\to 0} \sphericalangle(p_\gamma, p_i) \ \Rightarrow\ L$
- phase-space tuning s.t. $\cos\sphericalangle \sim x_i$

$\Rightarrow$ at most one small angle $\to$ FKS partitioning

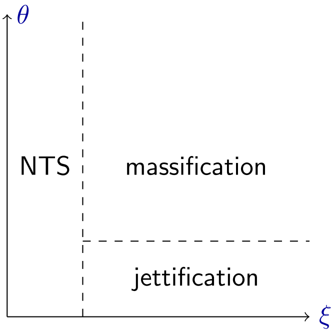


[Signer 22]

$ee \to \gamma^*$ can be taken to N^3 LO

- VVV: known
  [Fael, Lange, Schönwald, Steinhauser 22]
- RRR: "trivial"
- RRV: OpenLoops + NTS stabilisation
- RVV
  - massless [Bader, Kryś, Moodie, Zoia 2?] implemented, running at $\sim$ 130ev/s
  - maybe DiffExp

  $\Rightarrow$ LBK + jettification at two-loop



jettification

- expand for small emission angles

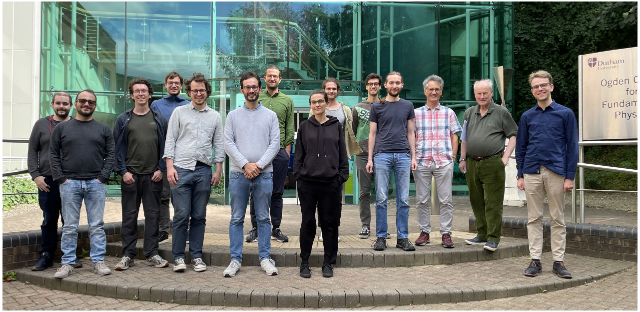
the NNLO era is here, not only for QCD, also for QED

future steps

- NNLO QED$\oplus$ EW
- NNLO QED$\oplus$ PS
- higher energies
- massification for real corrections
- collinear stabilisation

- N^3 LO for $\gamma^* \to \ell\ell$

$\Rightarrow$ Workstop in Durham



f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Durham), A.Gurgone (Pavia)
not shown: P.Banerjee (IIT Guwahati), Dnaiel Moreno (PSI), David Radic (Tubingen)

McMule

`mule-tools.gitlab.io`