University of Vienna

QED corrections for precision experiments

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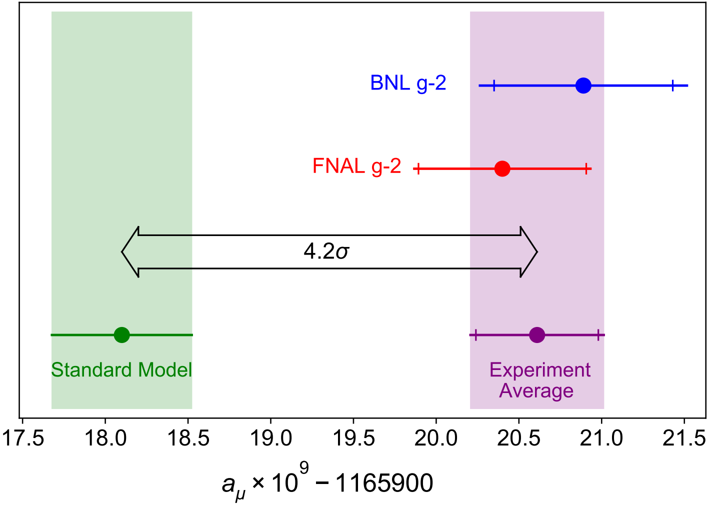
I hope to address the following

- $\alpha_{\text{QED}} \ll 1$, so why bother?

$\Rightarrow$ where do QED corrections matter?

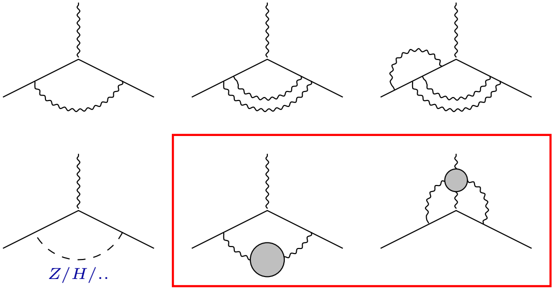
- what challenges?
- how to solve them (in pictures!)
- some phenomenology (more pretty pictures!)
- vision of the future

most precise measurement of $g-2$



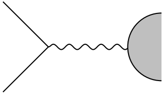
⇒ needs precise theory

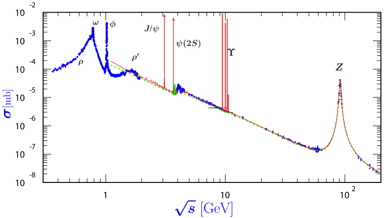
many Feynman diagrams, incl. non-perturbative



theory uncertainty from hadronic physics using optical theorem $s > 0$

$$\int \mathrm{d}s \Big(K(s) \quad \Big)$$

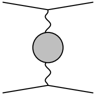




$\Rightarrow$ very messy!

using $t < 0$

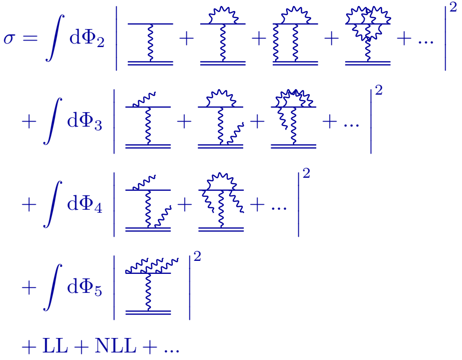
$$\int \mathrm{d}t \Big(K'(t) \quad \Big)$$



$\Rightarrow$ much cleaner but smaller

target accuracy: 10^{-5} ($\rightarrow$ 1% on HVP)

- dominant NNLO corr. with full m dep.
  [Carloni Calame et al. 20; Banerjee, Engel, Signer, YU 20]
- full NNLO corr. (currently w/o m^2/Q^2, add later) [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 2?]
- electronic $\mathrm{N^3LO}$ w/o m^2/Q^2
- resummation



the world is not just $g-2$...

- luminosity measurements $\Rightarrow$ $e^+e^- \to e^+e^-$ (Belle, FCC-ee, ...)
  [Banerjee, Engel, Schalch, Signer, YU 21]
- dark sector searches $\Rightarrow$ $e^+e^- \to \gamma\gamma$ (PADME, also for luminosity...)
  [Engel, Naterop, Signer, YU, Zoller 2?]
- R ratios $\Rightarrow$ $e^+e^- \to \mu^+\mu^-$ (DAΦNE, VEPP, ...)
- τ physics $\Rightarrow$ $e^+e^- \to \tau^+\tau^-$ (Belle) [Kollatzsch, YU 2?]
- proton radius $\Rightarrow$ $\ell p \to \ell p$ and $ee \to ee$ (P2, PRad, MUSE)
  [Bucoveanu, Spiesberger 18; Banerjee, Engel, Signer, YU 20; Banerjee, Engel, Schalch, Signer, YU 21]
- lepton decays $\Rightarrow$ $\ell \to \ell'\nu\bar{\nu} + \{ee, \gamma, \gamma\gamma\}$ (MEG, Mu3e, Belle, ...)
  [Pruna, Signer, YU 16; YU, 17; Engel, Gnendiger, Signer, YU, 18, Banerjee, Coutinho, Engel, Gurgone, Signer, YU 2?]

McMule

a framework for QED corrections

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18$^{\text{TH}}$ September 2022

basic strategy: use 40+ years of QCD experience on QED

- use automation where available and useful (eg. OpenLoops [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini 19])
- adapt QCD results where known (eg. [Bernreuther et al., 04])
- use methods invented (eg. [Frixione, Kunszt, Signer 96])

QED and QCD calculations have many common issues, but . . .

- Abelian structure $\Rightarrow$ a bit easier [no big deal]
- much simpler infrared structure [advantage]
- want/need $m \neq 0$ since $\log m$ physical [problem]
- more exclusive, e.g. hard collinear emission [problem]

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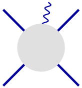
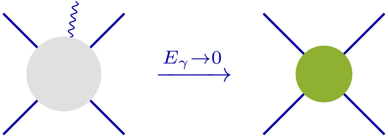
soft singularities

$$\int \mathrm{d}\Phi_\gamma$$  $$\sim \int_0 \mathrm{d}E_\gamma E_\gamma \int_{-1}^{1} \mathrm{d}(\cos\theta)\, \frac{1}{E_\gamma^2(1-\beta\cos\theta)}$$

$\Rightarrow$ luckily universality of soft singularities

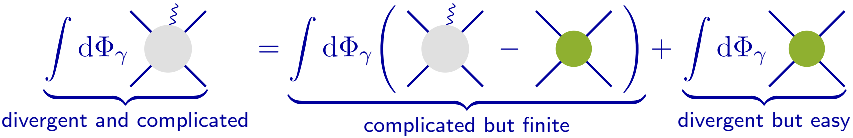


$$\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\,\mathcal{M}_n^{(\ell)} + \text{finite}$$

for any process and loop order. Similarly for virtual

$$e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \text{finite}$$

⇒ subtraction scheme at any order (FKS$^\ell$) [Engel, Signer, YU 19]



- very QCD-y
- based on [Frixione, Kunszt, Signer 96]
- no resolution parameter or photon mass, just DREG
- unphysical $0 < \xi_c \lesssim 1$ to test stability, implementation, ...

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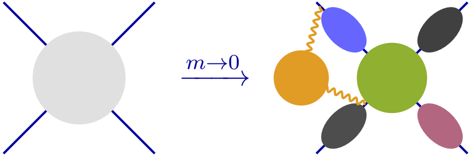
- loop integrals with internal masses are very complicated!
- but $m_e^2 \ll m_\mu^2 \sim Q^2$ for many applications

$\Rightarrow$ don't actually care about full m_e dependence

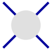
- but $\int\langle\text{expanded integrand}\rangle \neq \langle\text{expanded integral}\rangle$

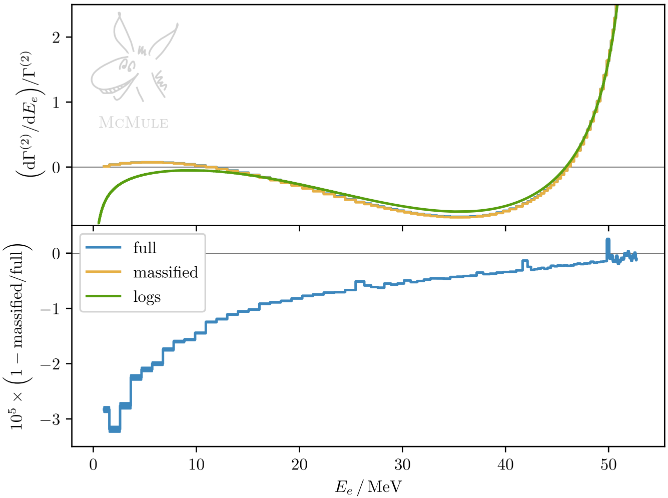
$\Rightarrow$ method of regions [Beneke, Smirnov 98] (hard, soft, collinear, ...)

universality of collinear singularities $\rightarrow$ calculate up to $\mathcal{O}(m^2/Q^2)$



$= S \times Z \times Z \times Z \times Z \times H$

- H: hard function $\sim$  $\Big|_{m=0}$
- Z: process independent jet function
- S: simple soft function



[Chen 18] v. [Engel, Gnendiger, Signer, YU 18] v. [Arbuzov et al. 02]

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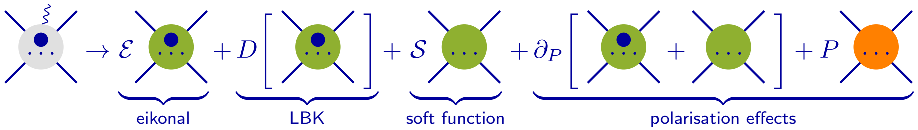
$$\mathcal{M}_{n+1}^{(\ell)} \sim \frac{1}{E_\gamma^2(1-\beta\cos\theta)}$$

real-virtual (or even real-real-virtual)

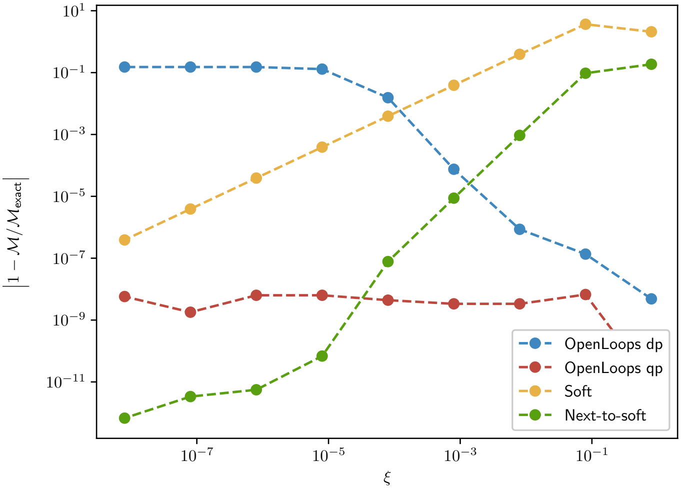
- 'trivial' in principle [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini et al. 19]
- extremely delicate numerically for $E_\gamma \to 0$ (or $\cos\theta \to 1$)

⇒ Taylor expand around $E_\gamma = 0$ if small

- LBK theorem [Low 58; Burnett, Kroll 67] and extension [Engel, Signer, YU 21]



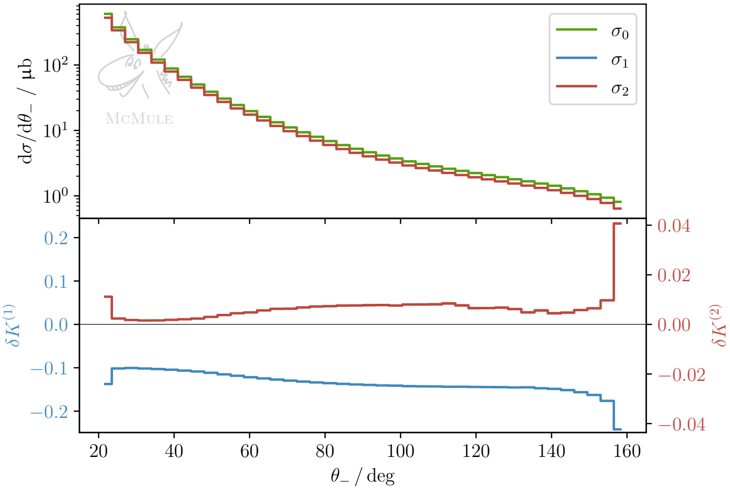
example $e^+e^- \to e^+e^-\gamma$ @ one-loop



compare with exact calculation in Mathematica

[Banerjee, Engel, Schalch, Signer, YU 21]

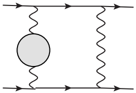
$\sqrt{s} = 1020\,\mathrm{MeV}$



$E_\pm > 408\,\mathrm{MeV},\ 20° \le \theta_\pm \le 160°,\ |180° - \theta_+ - \theta_-| < 10°$

a few more hurdles

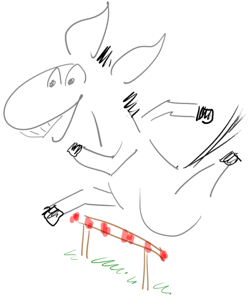
- VP diagrams for $e/\mu/\tau/$had$/\ldots$ numerically with full mass dependence



- collinear pseudo-singularities $\lim_{\to 0} \sphericalangle(p_\gamma, p_i) \;\Rightarrow\; L$
- phase-space tuning s.t. $\cos\sphericalangle \sim x_i$

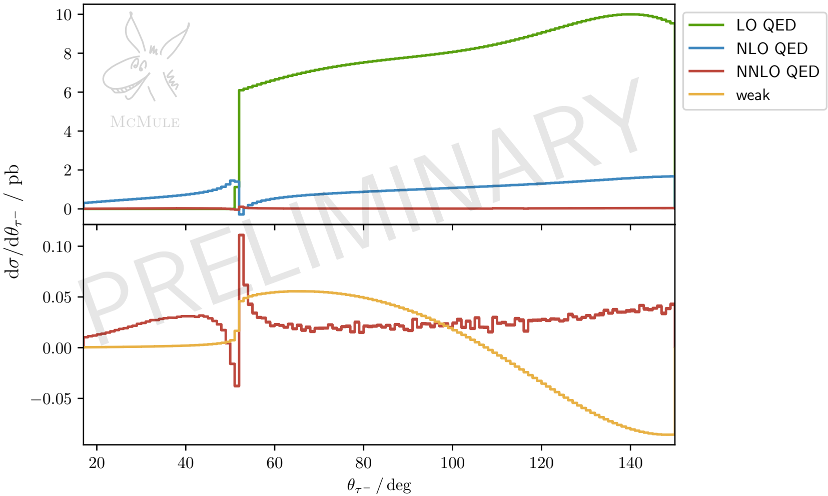
$\Rightarrow$ at most one small angle $\rightarrow$ FKS partitioning

- polarisation & EW where applicable

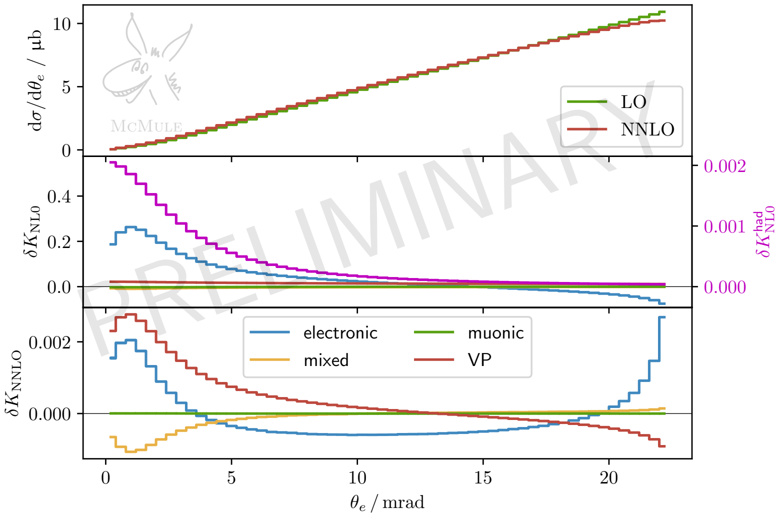


[Signer 22]

for Belle II, polarised initial state, NLO-EW⊕ dominant NNLO [Kollatzsch, YU 2?]



$E^{\mu}_{\text{beam}} = 150\,\text{GeV}$, $E_e > 1\,\text{GeV}$, $\theta_\mu > 0.3\,\text{mrad}$ [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 2?]



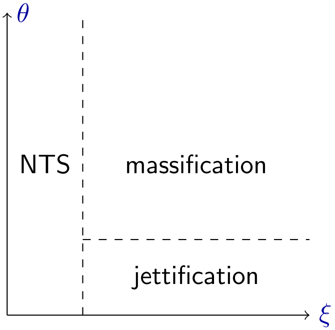
$ee \to \gamma^*$ can be taken to N^3 LO

- VVV: known
  [Fael, Lange, Schönwald, Steinhauser 22]
- RRR: "trivial"
- RRV: OpenLoops + NTS stabilisation
- RVV: massless known (three-jet @ NNLO), massive (DiffExp?)

$\Rightarrow$ LBK + jettification at two-loop

jettification

- expand for small emission angles



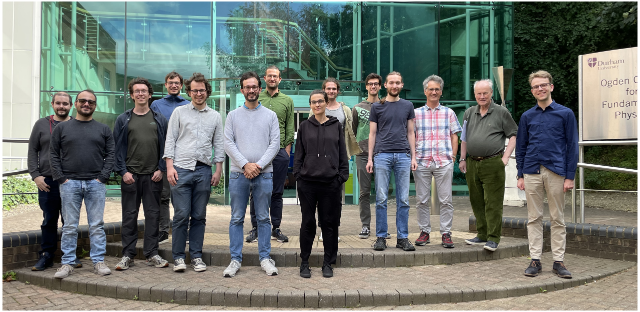
the NNLO era is here, not only for QCD, also for QED

future steps

- more NNLO QED⊕ EW
- NNLO QED⊕ PS
- higher energies
- massification for real corrections
- collinear stabilisation

- N^3 LO for $\gamma^* \to \ell\ell$

$\Rightarrow$ Workstop in Durham



f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC Valencia), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel ($\to$ Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Durham), A.Gurgone (Pavia), not shown: P. Banerjee (Zhejiang), A. Proust (Lyon)

McMule

`mule-tools.gitlab.io`

LBK theorem, alternative graphics