

Power Expansions on the Lightcone: From Theory to Phenomenology

QED calculations at NNLO (and beyond)

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non-exhaustive list of experiments as implemented in MCMULE

process	experiment	physics motivation
$e\mu \rightarrow e\mu$	MUonE	HVP to $(g - 2)_\mu$
$lp \rightarrow lp$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge normalisation
$e^-e^- \rightarrow e^-e^-$	Prad 2	$\sin^2 \theta_W$ at low Q^2
$e^+e^- \rightarrow e^+e^-$	MOLLER, ...	luminosity measurement
$ee \rightarrow ll$	any e^+e^- collider	R -ratio
$ee \rightarrow \gamma\gamma$	VEPP, BES, Daphne, ...	τ properties
	Belle	dark searches
	Daphne	luminosity measurement
	any e^+e^- collider	
$\mu \rightarrow \nu\bar{\nu}e$	MEG	ALP searches
	DUNE	beam-line profiling

QCD @ LHC	\Leftrightarrow	QED @ low & medium energy	
non-abelian	\approx	abelian	matrix elements somewhat easier
non-abelian	\gg	abelian	IR structure much easier ①
massless fermions	\ll	massive fermions	loop amplitudes much harder ②
jets	$<$	exclusive w.r.t. collinear radiation	numerics harder $\supset \log(m^2/Q^2) \equiv L$ much harder for small masses ③

stealing from QCD

- master integrals (reduction and computation), automated tools, EFT methods
- use dimensional regularisation for IR singularities, not photon mass
- use subtraction method for phase-space integration, not slicing method
- for the future: match fixed-order result to parton shower

soft singularities exponentiate [Yennie, Frautschi, Suura 61]

- universal soft limit $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E} \mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f} = \text{finite}$


use this to construct an all-order subtraction scheme FKS^ℓ

- nothing complicated needed higher than $\mathcal{O}(\epsilon^0)$
- only one universal CT: $\hat{\mathcal{E}}$

$$\underbrace{\int d\Phi_\gamma \text{ (grey blob) }}_{\text{divergent and complicated}} = \underbrace{\int d\Phi_\gamma \left(\text{grey blob} - \text{green blob} \right)}_{\text{complicated but finite}} + \underbrace{\int d\Phi_\gamma \text{ (green blob) }}_{\text{divergent but easy}}$$

universality of collinear singularities \rightarrow calculate up to $\mathcal{O}(m^2/Q^2)$



- H : hard function \sim  $\Big|_{m=0}$
- Z : process independent jet function
- S : simple soft function

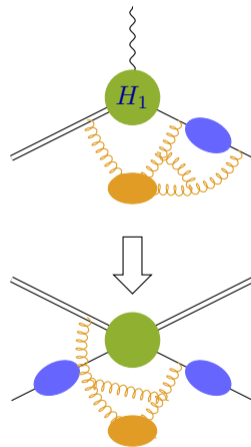
simple process ($\mu \rightarrow e\nu\nu$ or $t \rightarrow b\nu$)

- $\mathcal{A}_\mu(m) = \mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m)$
- $Z \supset \log(m)$: process indep. jet fct.
- $\mathcal{S} \supset \log(m)$: process dep. soft fct. (easy)

[Penin 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]

different process ($\mu e \rightarrow \mu e$)

- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$



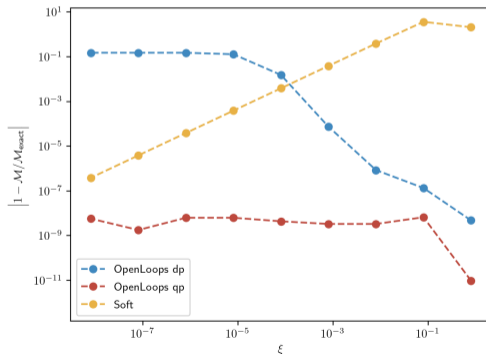
real-virtual corrections trivial in principle, extremely delicate numerically



example $ee \rightarrow ee\gamma$

[Banerjee, Engel, Schalch, Signer, YU 21]

- soft limit (of collinear emission)
 $E_\gamma = \xi \sqrt{s}/2$
- arbitrary prec. calculation vs **dp**, **qp**, **eikonal**, **NTS**
- **stability problem**



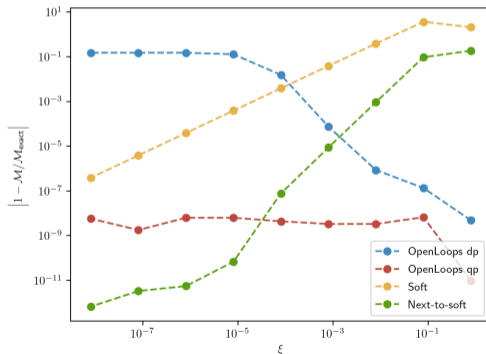
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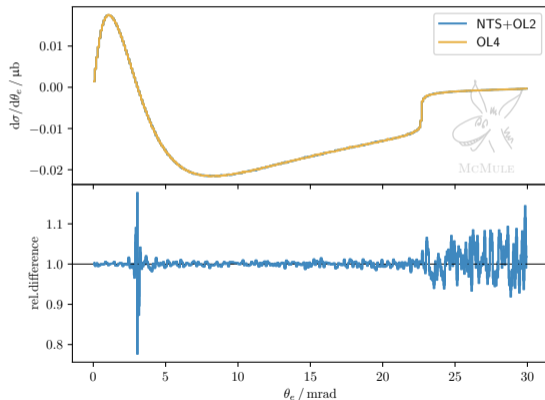
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- soft limit (of collinear emission)
 $E_\gamma = \xi \sqrt{s}/2$
- arbitrary prec. calculation vs **dp**, **qp**, **eikonal**, **NTS**
- stability problem solved & speed-up



test next-to-soft stabilisation vs OL4 (OpenLoops quad) for $\mu e \rightarrow \mu e$ real-virtual

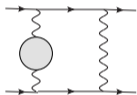


- same statistics, same result
 - 70 days vs 4 days
 - integrated results for different cuts
- ⇒ this is **not** an approximation but a numerical tool

NTS	OL4
-0.29268(4)	-0.29267(4)
-0.44789(6)	-0.44778(6)
-0.64662(9)	-0.64649(9)

a few more hurdles

- VP diagrams for $e/\mu/\tau/had/...$ numerically with full mass dependence



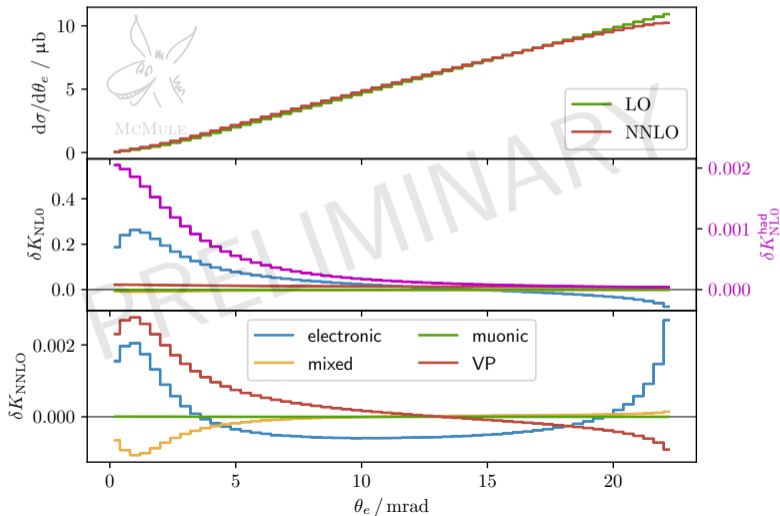
- collinear pseudo-singularities $\lim_{\rightarrow 0} \angle(p_\gamma, p_i) \Rightarrow L$
- phase-space tuning s.t. $\cos \angle \sim x_i$

\Rightarrow at most one small angle \rightarrow FKS partitioning



[Signer 22]

$E_{\text{beam}}^\mu = 150 \text{ GeV}$, $E_e > 1 \text{ GeV}$, $\theta_\mu > 0.3 \text{ mrad}$ [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 2?]

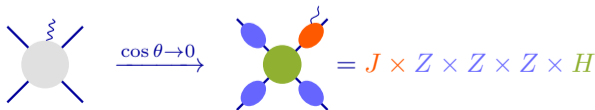
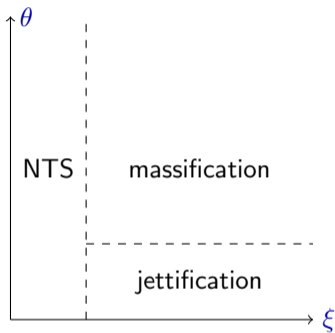


$ee \rightarrow \gamma^*$ can be taken to N³ LO

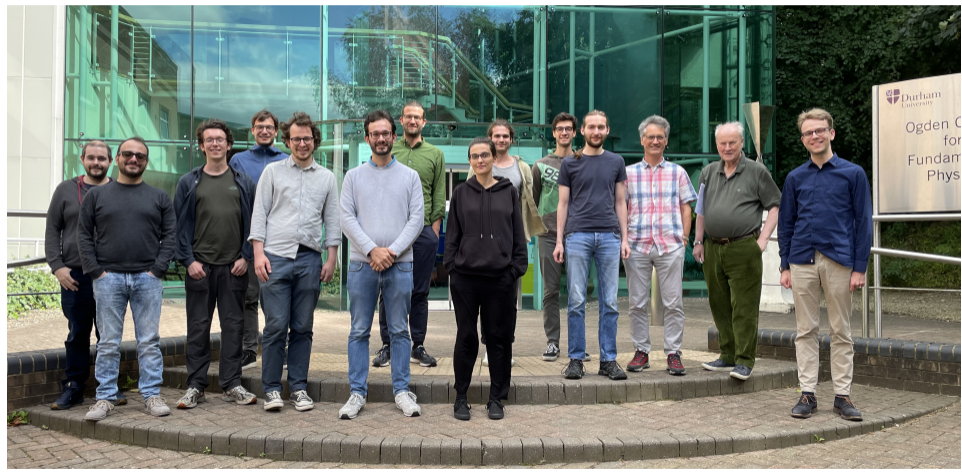
- VVV: known
[Fael, Lange, Schönwald, Steinhauser 22]
 - RRR: “trivial”
 - RRV: OpenLoops + NTS stabilisation
 - RVV: massless known (three-jet @ NNLO), massive (DiffExp?)
- ⇒ LBK + jettification at two-loop

jettification

- expand for small emission angles



N^3 LO workstop in Durham





McMULE

mule-tools.gitlab.io

f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (PSI), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (→ Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Durham), A.Gurgone (Pavia)