(**HP**)^2 **2022**

NNLO calculations for low-energy experiments

Yannick Ulrich

IPPP, University of Durham

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non-exhaustive list of experiments as implemented in McMule

process	experiment	physics motivation
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$
$\ell p \to \ell p$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge
$e^-e^- \to e^-e^-$	Prad 2	normalisation
	MOLLER, ...	$\sin\theta_W^2$ at low Q^2
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement
$ee \to \ell\ell$	VEPP, BES, Daphne, ...	R-ratio
	Belle	τ properties
$ee \to \gamma\gamma$	Daphne	dark searches
	any e^+e^- collider	luminosity measurement
$\mu \to \nu\bar\nu e$	MEG	ALP searches
	DUNE	beam-line profiling

QCD @ LHC	$\Leftrightarrow$	QED @ low & medium energy	
non-abelian	$\gtrsim$	abelian	matrix elements somewhat easier
non-abelian	$\gg$	abelian	IR structure much easier ①
massless fermions	$\ll$	massive fermions	loop amplitudes much harder ②
jets	$<$	exclusive w.r.t.	numerics harder $\supset \log(m^2/Q^2) \equiv L$
		collinear radiation	much harder for small masses ③

stealing from QCD

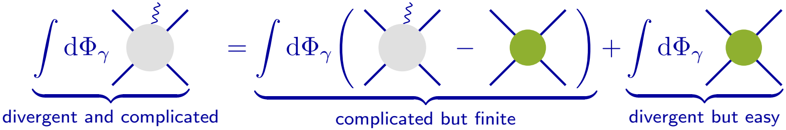
- master integrals (reduction and computation), automated tools, EFT methods
- use dimensional regularisation for IR singularities, not photon mass
- use subtraction method for phase-space integration, not slicing method
- for the future: match fixed-order result to parton shower

soft singularities exponentiate [Yennie, Frautschi, Suura 61]

- universal soft limit $\mathcal{M}^{(\ell)}_{n+1} = \mathcal{E}\mathcal{M}^{(\ell)}_n + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}^{(\ell)}_n = \sum_{\ell=0}^{\infty} \mathcal{M}^{(\ell)f}_n =$ finite

use this to construct an all-order subtraction scheme FKS$^\ell$

- nothing complicated needed higher than $\mathcal{O}(\epsilon^0)$
- only one universal CT: $\hat{\mathcal{E}}$



masses are physical in QED $\Rightarrow$ keep masses

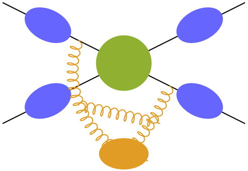
- drop polynomially suppressed terms at two-loop $\to$ error $\sim \left(\frac{\alpha}{\pi}\right)^2 \log\frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$
- based on factorisation, SCET, and method of regions

  [Penin 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g. $ee \to ee$ at two-loop:

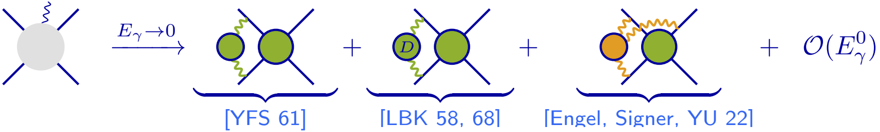
$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$

- soft: process-dependent $S = 1 +$ fermion loops

  $\to$ compute separately anyway to combine with hadron loops
- collinear: universal Z, converts $1/\epsilon \to \log(m^2/Q^2)$
- hard: massless calculation



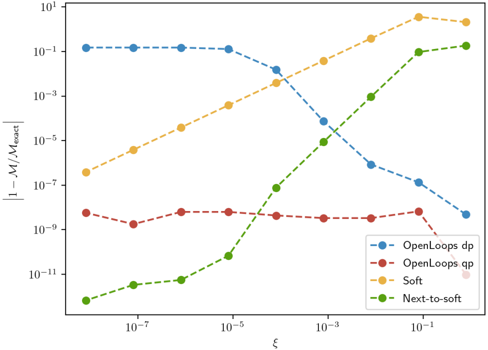
real-virtual corrections trivial in principle, extremely delicate numerically



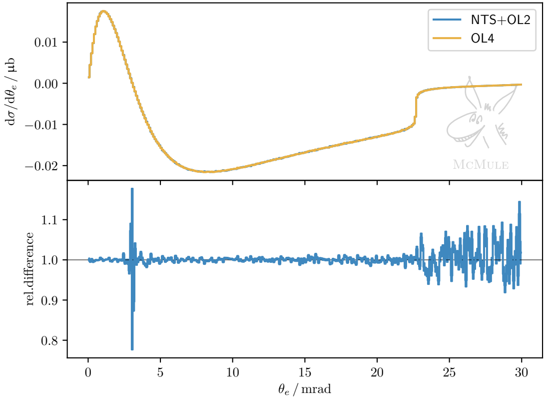
example $ee \to ee\gamma$

[Banerjee, Engel, Schalch, Signer, YU 21]

- soft limit (of collinear emission) $E_\gamma = \xi\sqrt{s}/2$
- arbitrary prec. calculation vs dp, qp, eikonal, NTS
- stability problem solved & speed-up



test next-to-soft stabilisation vs OL4 (OpenLoops quad) for $\mu e \to \mu e$ real-virtual



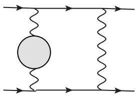
- same statistics, same result
- 70 days vs 4 days
- integrated results for different cuts

⇒ this is not an approximation but a numerical tool

NTS	OL4
$-0.29268(4)$	$-0.29267(4)$
$-0.44789(6)$	$-0.44778(6)$
$-0.64662(9)$	$-0.64649(9)$

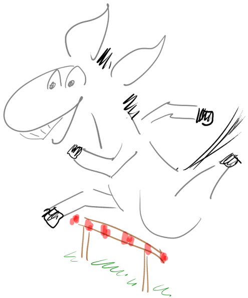
a few more hurdles

- VP diagrams for $e/\mu/\tau/$had$/\ldots$ numerically with full mass dependence



- collinear pseudo-singularities $\lim_{\to 0} \sphericalangle(p_\gamma, p_i) \;\Rightarrow\; L$
- phase-space tuning s.t. $\cos\sphericalangle \sim x_i$

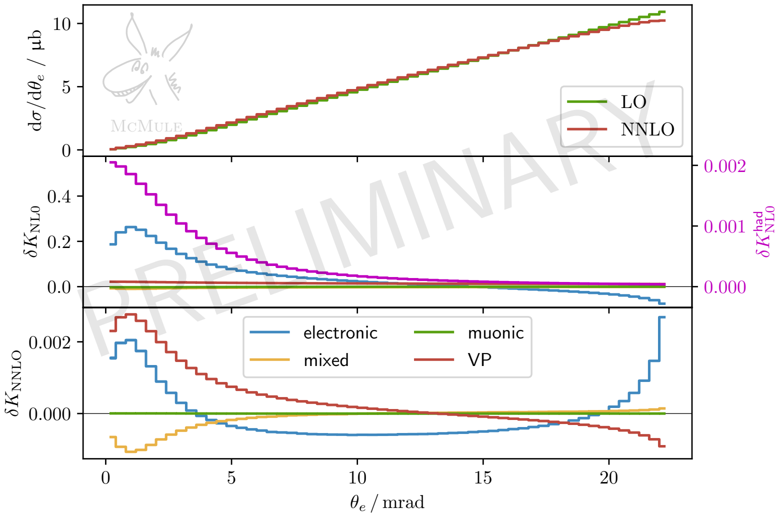
$\Rightarrow$ at most one small angle $\to$ FKS partitioning



[Signer 22]

pheno example: full NNLO for $e\mu \to e\mu$

$E^{\mu}_{\text{beam}} = 150\,\text{GeV}$, $E_e > 1\,\text{GeV}$, $\theta_\mu > 0.3\,\text{mrad}$ [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 2?]



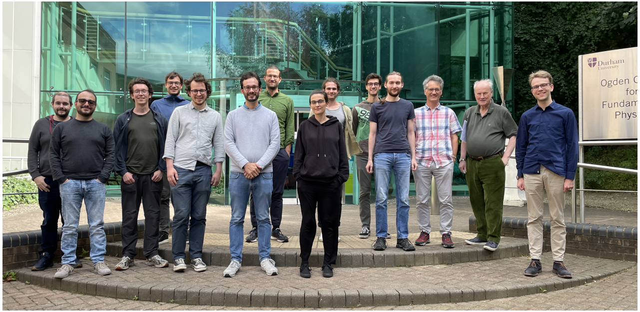
the NNLO era is here, not only for QCD, also for QED

future steps

- NNLO QED$\oplus$ EW
- NNLO QED$\oplus$ PS
- higher energies
- massification for real corrections
- collinear stabilisation

- N^3 LO for $\gamma^* \to \ell\ell$

$\Rightarrow$ Workstop in Durham



f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (PSI), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (→ Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Durham), A.Gurgone (Pavia)

McMule

`mule-tools.gitlab.io`