

ECT* Workshop

Radiative Corrections from medium to high energy experiments

NNLO results with McMule

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methods → Adrian Signer's talk

6+ processes implemented at NNLO

- $\mu \rightarrow \nu \bar{\nu} e$ (*) [Engel, Signer, YU 18]
- $e\mu \rightarrow e\mu$ (*) and $ee \rightarrow \mu\mu$ [Banerjee, Engel, Signer, YU 20], [Kollatzsch, YU 2?], [YU et al. 2?]
- $ep \rightarrow ep$ and $\mu p \rightarrow \mu p$ (#) [Banerjee, Engel, Signer, YU 20]
- $e\nu \rightarrow e\nu$ (#)
- $e^-e^- \rightarrow e^-e^-$ (*) and $e^+e^- \rightarrow e^+e^-$ (#) [Banerjee, Engel, Schalch, Signer, YU 21]
- $ee \rightarrow \gamma\gamma$ (#)
- + many more at NLO

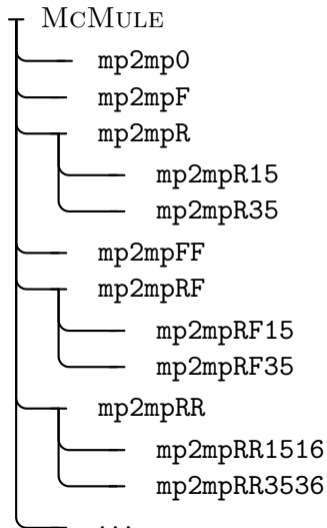
*: complete

#: complete except some hadronic effects

get the code <https://gitlab.com/mule-tools/mcmule>

- compile with `./configure && make`
 - define observables in `src/user.f95`
 - run all necessary pieces (SLURM script available)
 - ℓ - p scattering at NNLO
 - plot results with python
- ⇒ examples from **all** published results online

gitlab.com/mule-tools/user-library



example: $e(q_1)p(q_2) \rightarrow e(q_3)p(q_4)$ `user-library:l-p-scattering/mesa-legacy/ep2ep_mesa_paper/user.f95`

```
integer, parameter :: nr_q = 1
integer, parameter :: nr_bins = 200
real, parameter :: min_val(nr_q) = (/ 25. /)
real, parameter :: max_val(nr_q) = (/ 45. /)

FUNCTION QUANT(q1,q2,q3,q4,q5,q6,q7)

real(kind=prec) :: q1(4),q2(4),q3(4),q4(4),q5(4),q6(4),q7(4)
real(kind=prec), parameter :: ez = (/0.,0.,1.,0./)
real(kind=prec) :: quant(nr_q), thetae

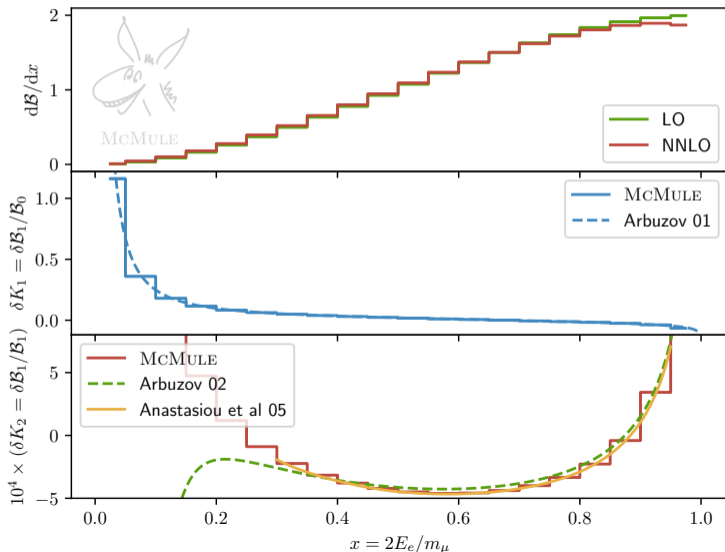
thetae = acos(cos_th(ez,boost_rf(q2, q3)))

pass_cut = .true.
if(thetae < 25*pi/180._prec) pass_cut=.false.
if(thetae > 45*pi/180._prec) pass_cut=.false.

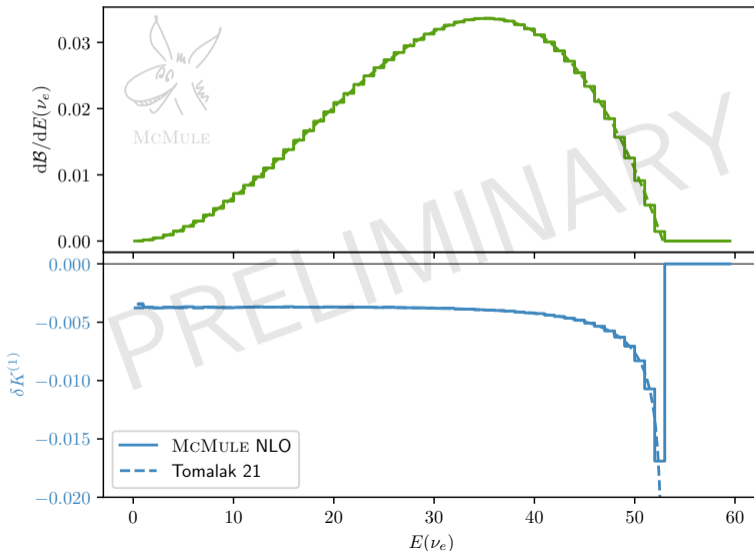
names(1) = "thetae"
quant(1) = 180*thetae/pi
END FUNCTION QUANT
```

- analytic approximations at NNLO $\mathcal{O}(\alpha^2 \log^{\{1,2\}}(m_e/m_\mu))$ [Arbuzov, Melnikov 02]
- numerical loop calculation
[Anastasiou, Melnikov, Petriello 05]
- first NNLO calculation in MCMULE to test FKS²
[Engel, Signer, YU 18]
- averaged over the neutrinos \Rightarrow currently redoing things to fix that
[Proust, Rocco, Signer, YU 2?]

mule-tools.gitlab.io/user-library/michel-decay/validation



mule-tools.gitlab.io/user-library/michel-decay/neutrino-spectrum



- largest uncertainty in $(g - 2)_\mu$ is HVP
- lattice or $e^+e^- \rightarrow$ hadrons data

new-ish proposal: MUonE

[Carloni Calame et al. 15, Abbiendi et al. 17, MUonE Letter of Intent 19]

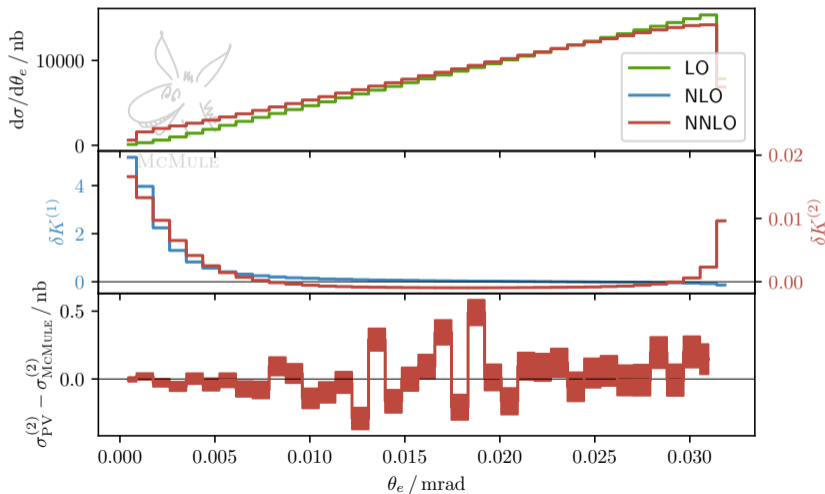
- 150 GeV muon beam (M1 @ CERN) on Be electrons
- measure $\theta_{e,\mu} \rightarrow$ re-construct $t < 0 \rightarrow \Delta\alpha_{\text{had}}(t) \rightarrow (g - 2)_\mu^{\text{HVP}}$
- target precision $10^{-5} \rightarrow \text{NNLO} \oplus \text{leading N}^3\text{LO} \oplus \text{PS} \oplus \dots$ [MUonE Theory Initiative 20]



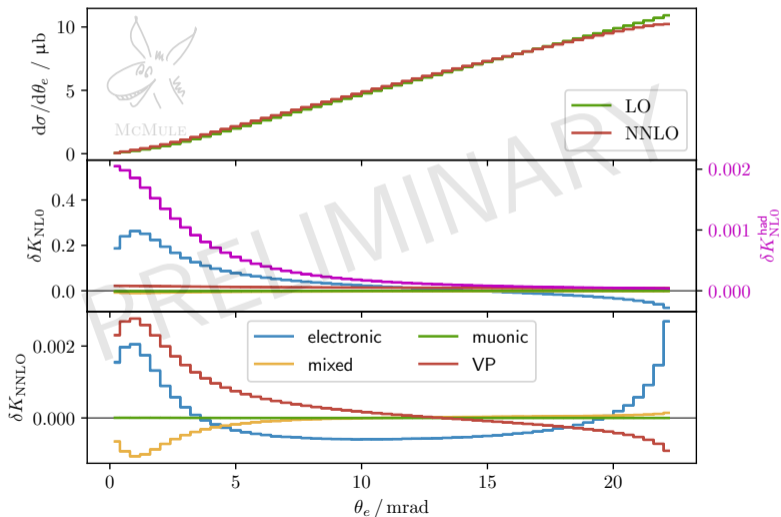
- full EW-NLO
[Alacevich, Carloni Calame, Chiesa, Montagna, Nicosini, Piccinini 18]
- hadronic corrections
[Fael 18, Fael, Passera 19, Balzani, Laporta, Passera 22]
- New Physics: contamination & searches
[Masiero, Paradisi, Passera 20, ...] [di Cortona, Nardi 22, ...]
- dominant NNLO \Rightarrow agreement!
[Carloni Calame, Chiesa, Hasan, Montagna, Nicosini, Piccinini 20] & [Banerjee, Engel, Signer, YU 20]
- pair-production and fermion loops
[Budassi, Carloni Calame, Chiesa, Del Pio, Hasan, Montagna, Nicosini, Piccinini 21 & 22]
- two-loop with $m = 0$
[Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, Torres Bobadilla, Tramontano 22]

compared with [Carloni Calame, Chiesa, Hasan, Montagna, Nicosini, Piccinini 20]

mule-tools.gitlab.io/user-library/mu-e-scattering/muone



massified $0 \lesssim m$, compared RV+RR [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 2?]



leptonic corrections for $e-p$ and $\mu-p$ scattering at NNLO

[Banerjee, Engel, Signer, YU 20]

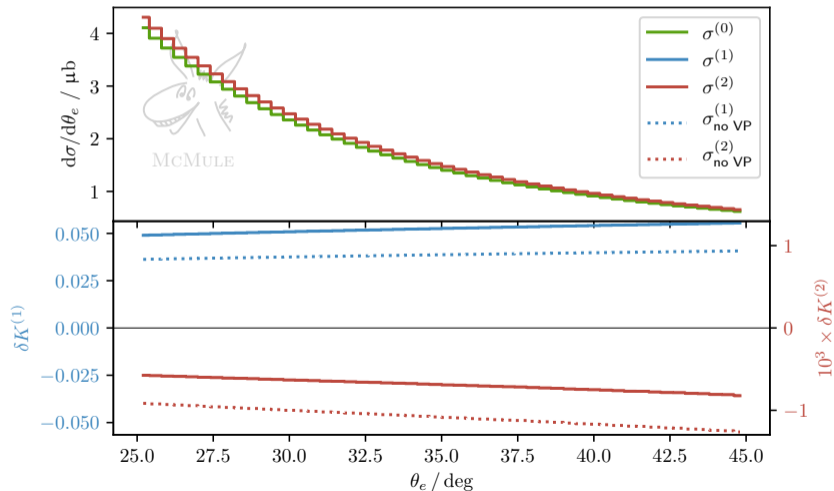
- parametrise proton as form factor $\gamma_\mu \rightarrow F_1 \gamma^\mu + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{2m}$
- part of dominant $\mu-e$ scattering \Rightarrow very well checked!
- discrepancy with previous calculation [Bucoveanu, Spiesberger 18]
 - **but** agree on matrix elements
 - integration the same as $\mu-e$ scattering
- trivial to switch $e-p$ to $\mu-p$ scattering $F_2 \rightarrow 0$
- **examples for**
 MESA ($p_e = 155 \text{ MeV}$) and MUSE ($p_\mu = 210 \text{ MeV}$)

TPE contribution

- WIP [Engel, Hagelstein, Rocco, Sharkovska, YU ??]

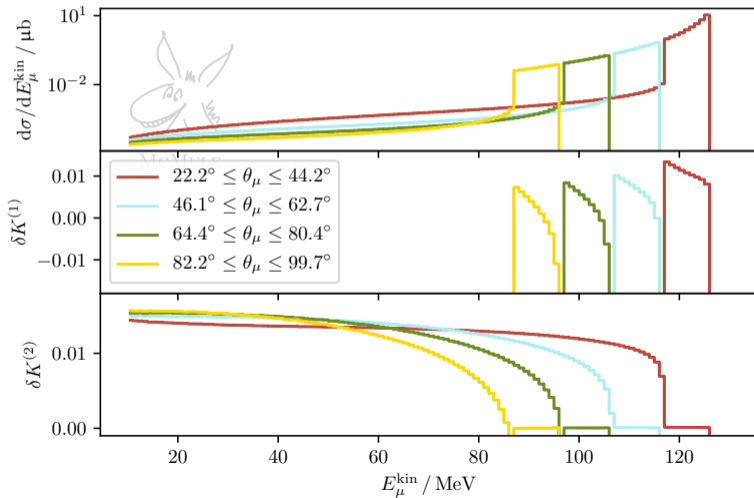
$p_e^{\text{in}} = 155 \text{ MeV}, E_e^{\text{out}} > 45 \text{ MeV}$

mule-tools.gitlab.io/user-library/l-p-scattering/mesa-legacy



$p_{\mu}^{\text{in}} = 210 \text{ MeV}, 20^{\circ} < \theta_{\mu} < 100^{\circ}$

mall mule-tools.gitlab.io/user-library/l-p-scattering/muse-legacy



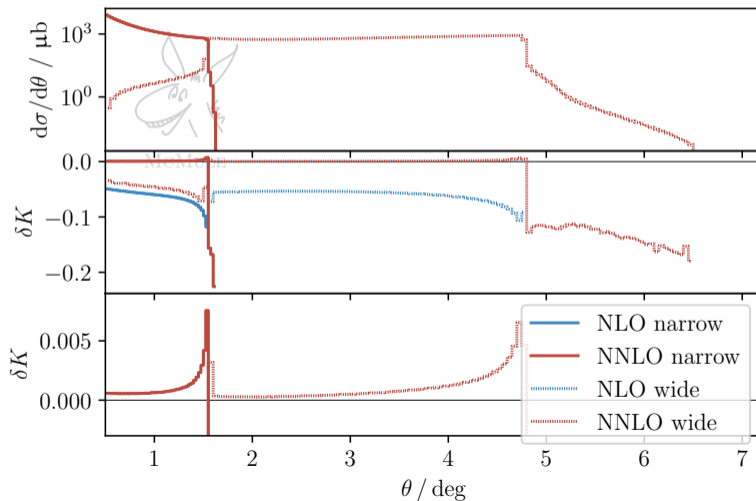
- used for normalisation by PRad ($E = 1.4 \text{ GeV}$)

$$\left(\frac{d\sigma}{d\theta}\right)_{ep} = \left[\frac{N_{\text{exp}}(ep \rightarrow ep, \theta_i)}{N_{\text{exp}}(ee \rightarrow ee, \theta_i)}\right] \left(\frac{d\sigma}{d\theta}\right)_{ee}$$

- massless two-loop known [Bern, Dixon, Ghinculov 00]
- NLO calculations [Kaiser 10, Akushevich, Gao, Ilyichev, Meziane 15], and generators [Afanasev, Chudakov, Ilyichev, Zykunov 06, Epstein, Milner 16]
- NNLO with MCMULE using NTS stabilisation & massification [Banerjee, Engel, Schalch, Signer, YU 21]
- validated by comparing $e^+e^- \rightarrow e^+e^-$ to Babayaga (NLO \oplus PS) [Carloni Calame, Montagna, Nicosini, Piccinini 04]
- two indistinguishable particles \Rightarrow 'narrow' and 'wide' electron

$0.5^\circ < \theta_i < 6.5^\circ$, $\Delta E < 130 \text{ MeV}$, $\Delta\phi < 7.35^\circ$

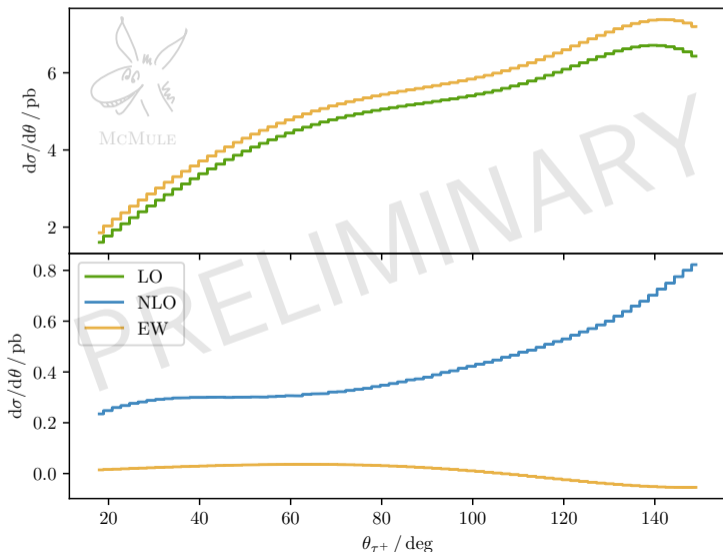
mule-tools.gitlab.io/user-library/moller-scattering/prad



Miscellaneous

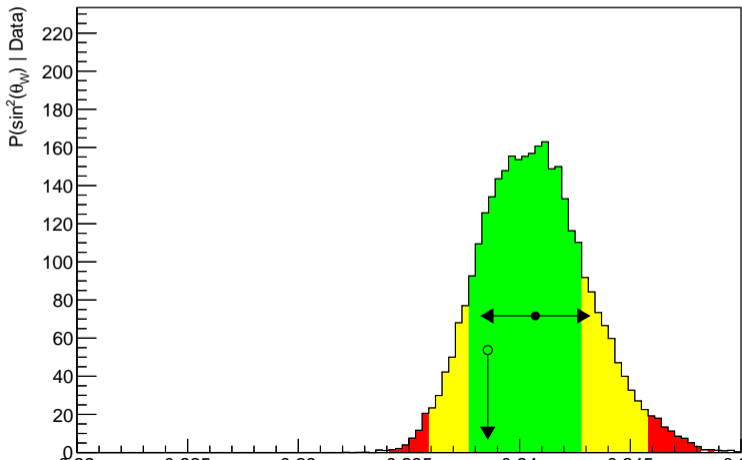
- $ee \rightarrow \tau\tau$ EW-NLO [Kollatzsch, YU 2?]
- $e\nu \rightarrow e\nu$ at NLO used for event generation
- best possible theory spectrum for $\mu \rightarrow \nu\bar{\nu}e$

$\sqrt{s} = m_{\Upsilon(4s)} = 10.5 \text{ GeV} \Rightarrow \text{EW effects}$

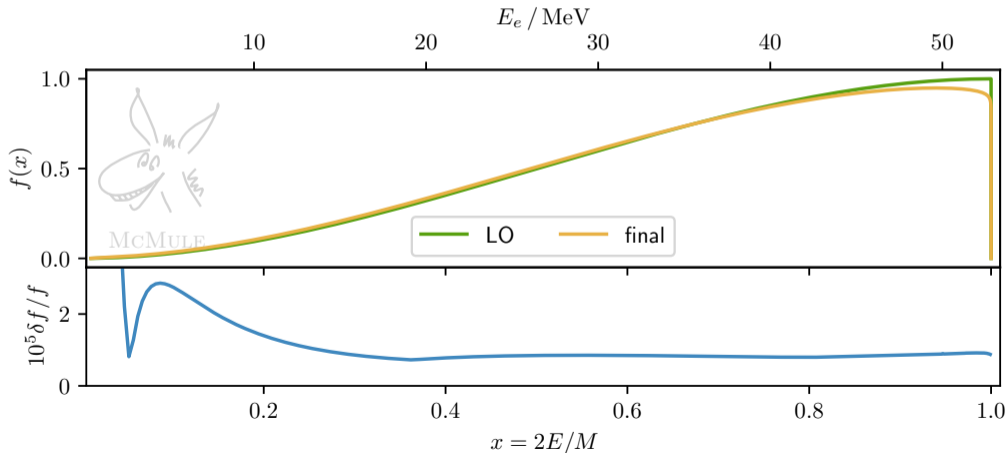


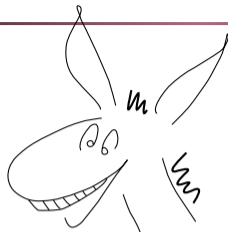
$\sin^2 \theta_W$ @ DUNE [de Gouvêa, Machado, Perez-Gonzalez, Tabrizi 19]

- what is the best observable?
- what influence do flux uncertainties and radiative corrections have? \Rightarrow full (N)NLO



- ALPs in the muon decay $\mu \rightarrow e X$
- \Rightarrow need $\mathcal{O}(10^{-5})$ at end point of E_e if $m_X \ll m_\mu$
- large logarithms due to soft-photons \Rightarrow resummation





McMULE

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