University of Sussex

QED corrections for precision experiments

Yannick Ulrich

IPPP, University of Durham

28$^{\text{TH}}$ February 2022

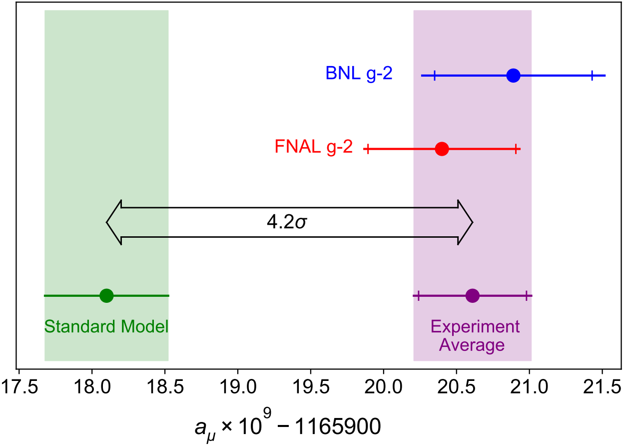
I hope to address the following

- $\alpha_{\mathrm{QED}} \ll 1$, so why bother?

$\Rightarrow$ where do QED corrections matter?

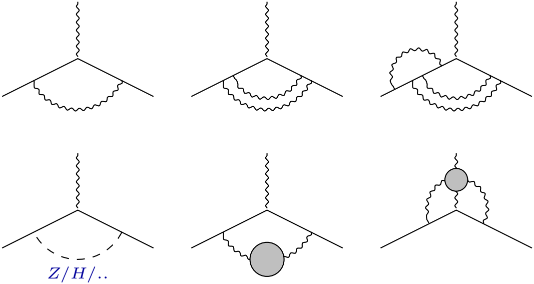
- what challenges?
- how to solve them (in pictures!)
- some phenomenology (more pretty pictures!)
- vision of the future

most precise measurement of $g-2$



$\Rightarrow$ needs precise theory

many Feynman diagrams, incl. non-perturbative



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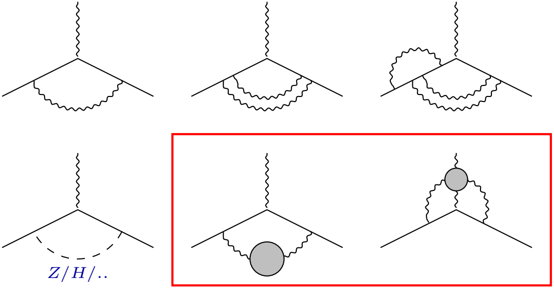
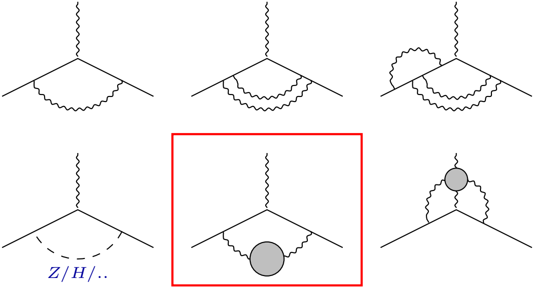


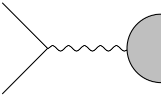
leading theory uncertainty

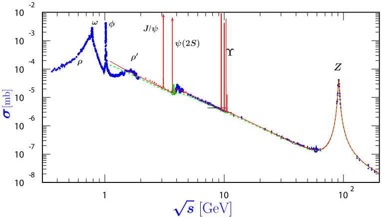
many Feynman diagrams, incl. non-perturbative



the hadronic vacuum polarisation

using optical theorem $s > 0$

$$\int \mathrm{d}s \Big(K(s)$$  $$\Big)$$



$\Rightarrow$ very messy!

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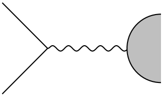
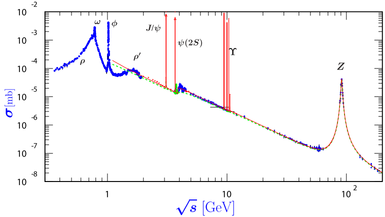
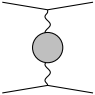
$$\int \mathrm{d}s \Big(K(s)$$  $$\Big)$$



$\Rightarrow$ very messy!

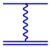
using $t < 0$

$$\int \mathrm{d}t \Big(K'(t)$$  $$\Big)$$

$\Rightarrow$ much cleaner but smaller

target accuracy: 10^{-5} ($\rightarrow$ 1% on HVP)

- dominant NNLO corr. with full m dep.
  [Carloni Calame et al. 20; Banerjee, Engel, Signer, YU 20]

$\sigma = \int \mathrm{d}\Phi_2 \,\Big|$  $+ \dots \Big|^2$

- full NNLO corr. w/o m^2/Q^2
- electronic $\mathrm{N^3LO}$ w/o m^2/Q^2
- resummation

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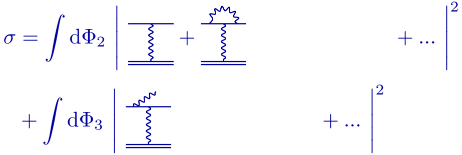

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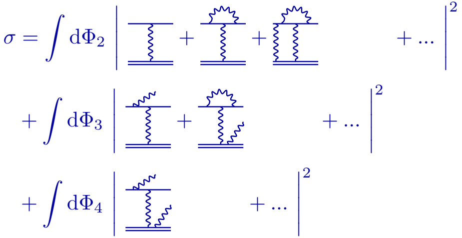

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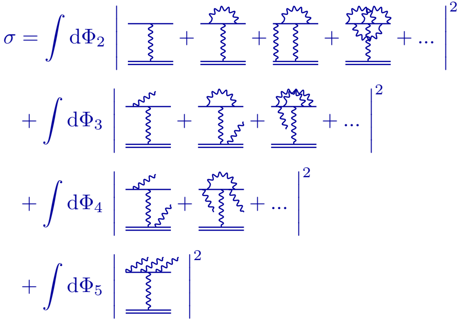

$$+\,\mathrm{LL} + \mathrm{NLL} + \ldots$$

the world is not just $g-2$...

- luminosity measurements ⇒ $e^+e^- \to e^+e^-$ (Belle, FCC-ee, ...)
  [Banerjee, Engel, Schalch, Signer, YU 21]
- dark sector searches ⇒ $e^+e^- \to \gamma\gamma$ (PADME, also for luminosity...)
  [Engel, Naterop, Signer, YU, Zoller 2?]
- R ratios ⇒ $e^+e^- \to \mu^+\mu^-$ (DAΦNE, VEPP, ...)
- τ physics ⇒ $e^+e^- \to \tau^+\tau^-$ (Belle)
  [Kollatzsch, YU 2?]
- proton radius ⇒ $\ell p \to \ell p$ and $ee \to ee$ (P2, PRad, MUSE)
  [Bucoveanu, Spiesberger 18; Banerjee, Engel, Signer, YU 20; Banerjee, Engel, Schalch, Signer, YU 21]
- lepton decays ⇒ $\ell \to \ell' \nu\bar\nu + \{ee, \gamma, \gamma\gamma\}$ (MEG, Mu3e, Belle, ...)
  [Pruna, Signer, YU 16; YU, 17; Engel, Gnendiger, Signer, YU, 18]

the real title page

McMule

a framework for QED corrections

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basic strategy: use 40+ years of QCD experience on QED

- use automation where available and useful (eg. OpenLoops [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini 19])
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QED and QCD calculations have many common issues, but . . .

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- Abelian structure $\Rightarrow$ a bit easier [no big deal]
- much simpler infrared structure [advantage]
- want/need $m \neq 0$ since $\log m$ physical [problem]
- more exclusive, e.g. hard collinear emission [problem]

QED vs. QCD

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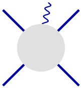
IR structure

soft singularities

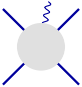
$$\int \mathrm{d}\Phi_\gamma$$



$$\sim \int_0 \mathrm{d}E_\gamma E_\gamma \int_{-1}^{1} \mathrm{d}(\cos\theta)\ \frac{1}{E_\gamma^2(1-\beta\cos\theta)}$$

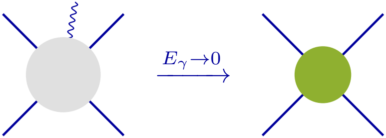
soft singularities

$$\int \mathrm{d}\Phi_\gamma$$



$$\sim \int_0 \mathrm{d}E_\gamma E_\gamma \int_{-1}^{1} \mathrm{d}(\cos\theta)\; \frac{1}{E_\gamma^2(1-\beta\cos\theta)}$$

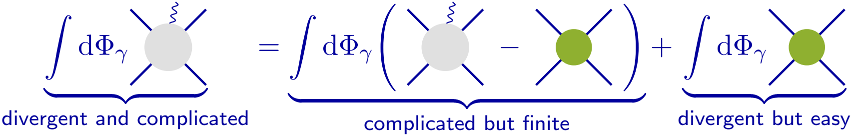
$\Rightarrow$ luckily universality of soft singularities



$$\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\,\mathcal{M}_n^{(\ell)} \; + \; \text{finite}$$

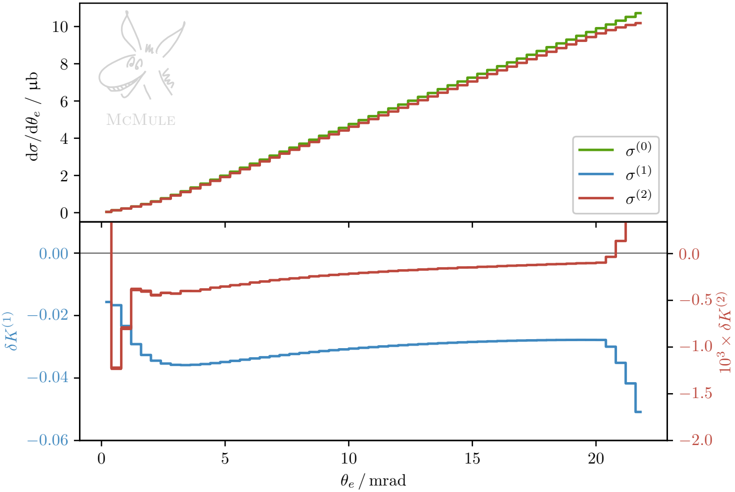
for any process and loop order

$\Rightarrow$ subtraction scheme at any order (FKS$^\ell$) [Engel, Signer, YU 19]



- very QCD-y
- based on [Frixione, Kunszt, Signer 96]
- no resolution parameter or photon mass, just DREG
- unphysical $0 < \xi_c \lesssim 1$ to test stability, implementation, ...

$E_\mu^{\text{beam}} = 150\ \text{GeV}$ with $E_e > 1\ \text{GeV}$, $\theta_\mu > 0.3\ \text{mrad}$, $0.9 < \theta_\mu/\theta_\mu^{\text{el}}(\theta_e) < 1.1$



[Banerjee, Engel, Signer, YU 20] using [Bernreuther et al. 04; Fael 18], checked [Carloni Calame et al. 20]

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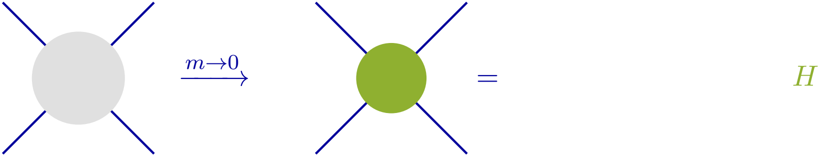
- loop integrals with internal masses are very complicated!
- but $m_e^2 \ll m_\mu^2 \sim Q^2$ for many applications

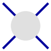
$\Rightarrow$ don't actually care about full m_e dependence

- but $\int \langle\text{expanded integrand}\rangle \neq \langle\text{expanded integral}\rangle$

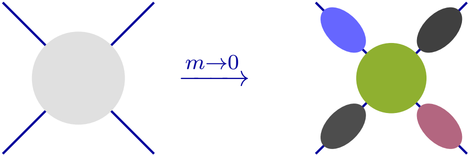
$\Rightarrow$ method of regions [Beneke, Smirnov 98] (hard, soft, collinear, ...)

universality of collinear singularities $\rightarrow$ calculate up to $\mathcal{O}(m^2/Q^2)$

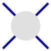


- H: hard function $\sim$  $\Big|_{m=0}$

universality of collinear singularities → calculate up to $\mathcal{O}(m^2/Q^2)$

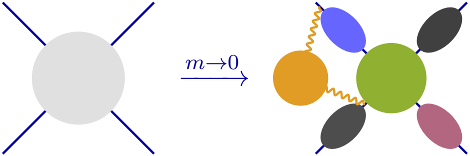


$= \quad Z \times Z \times Z \times Z \times H$

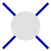
- H: hard function $\sim$  $\big|_{m=0}$
- Z: process independent jet function

SCET-ish IR factorisation

universality of collinear singularities $\rightarrow$ calculate up to $\mathcal{O}(m^2/Q^2)$



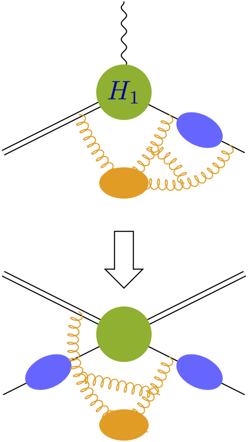
$$= S \times Z \times Z \times Z \times Z \times H$$

- H: hard function $\sim$  $\Big|_{m=0}$
- Z: process independent jet function
- S: simple soft function

simple process ($\mu \to e\nu\nu$ or $t \to b\ell\nu$)

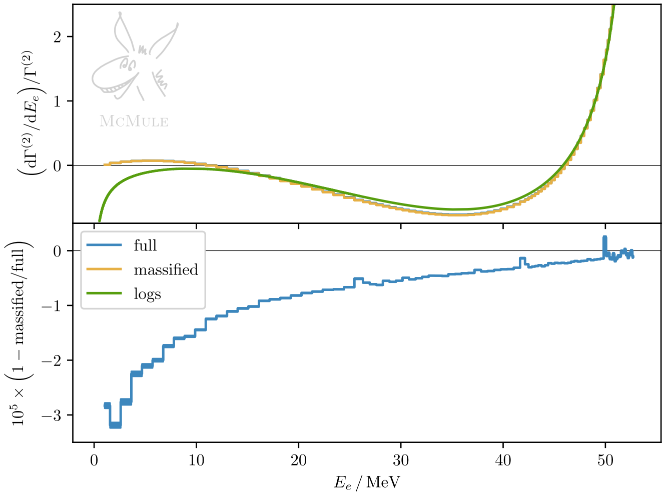
- $\mathcal{A}_\mu(m) = \mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m)$
- $Z \supset \log(m)$: process indep. jet fct.
- $\mathcal{S} \supset \log(m)$: process dep. soft fct. (easy)

[Penin 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]



different process ($\mu e \to \mu e$)

- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$



[Chen 18] v. [Engel, Gnendiger, Signer, YU 18] v. [Arbuzov et al. 02]

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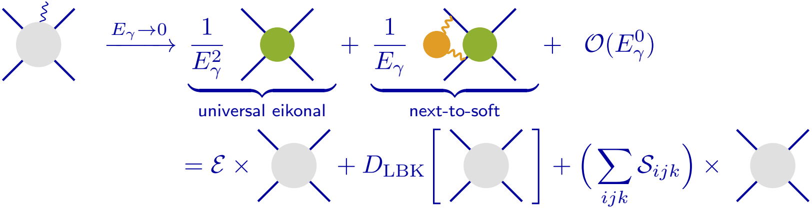
numerical stability

real-virtual (or even real-real-virtual) $\mathcal{M}^{(\ell)}_{n+1} \sim \frac{1}{E_\gamma^2(1-\beta\cos\theta)}$

- 'trivial' in principle [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini et al. 19]
- extremely delicate numerically for $E_\gamma \to 0$ (or $\cos\theta \to 1$)

$\Rightarrow$ Taylor expand around $E_\gamma = 0$ if small



- LBK theorem [Low 58; Burnett, Kroll 67] and extension [Engel, Signer, YU 21]

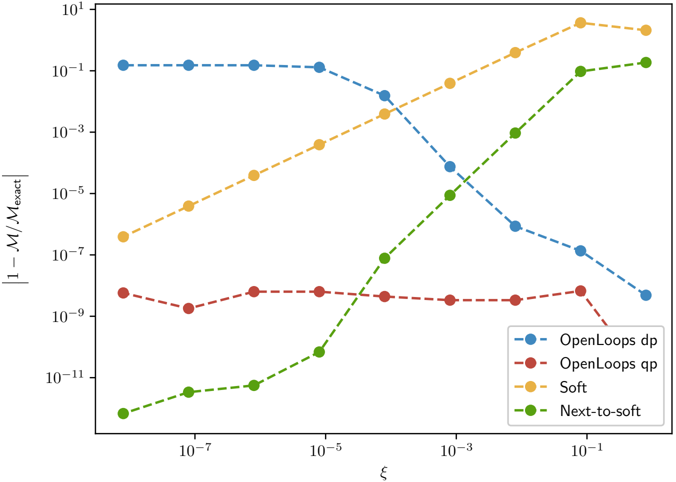
$\mathcal{E}$ and $\mathcal{S}$ are multiplications, what about $D_{\rm LBK}$?

$$D_{\rm LBK} = \frac{p_j^\mu}{k\cdot p_j}\left[\frac{p_i^\mu\,k^\nu}{k\cdot p_i} - g^{\mu\nu}\right]\frac{\partial}{\partial p_i^\nu}$$

- have to take deriviative wrt. external momenta
- usually done analytically
- can be done numerically as well using phase space routine `PS` $:\ \vec{x}\to\{p_i\}$ using

$$\frac{\partial}{\partial p_i^\nu} = \frac{\partial s_{jk}}{\partial p_i^\nu}\left[\frac{\partial s_{jk}}{\partial x_l}\right]^{-1}\frac{\partial}{\partial x_l}$$

- naive numerical derivative ($h=10^{-8}$) in McMule example $e^+e^- \to e^+e^-\gamma$ @ one-loop

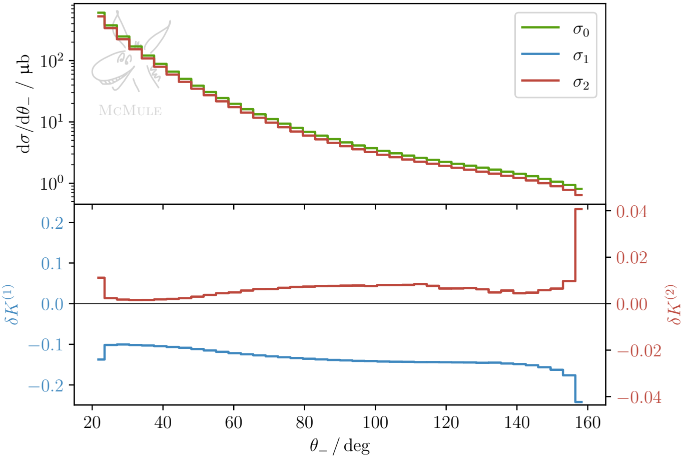


compare with exact calculation in Mathematica

[Banerjee, Engel, Schalch, Signer, YU 21]

pheno III: $e^+e^- \to e^+e^-$

$\sqrt{s} = 1020\,\mathrm{MeV}$



$E_\pm > 408\,\mathrm{MeV},\ 20^\circ \le \theta_\pm \le 160^\circ,\ \left|180^\circ - \theta_+ - \theta_-\right| < 10^\circ$

visions for the future

- polarisation effects ($ee \to \tau\tau$, $\ell p \to \ell p$, ...)
- EW effects ($ee \to \tau\tau$, $\ell p \to \ell p$, $ee \to ee$, $ee \to \gamma\gamma$, ...)
- finish NNLO for μ-e scattering
- HLbL in $ee \to \gamma\gamma$ & TPE in $\ell p \to \ell p$
- higher orders ($\mathrm{N^3LO}$ for $e\mu \to e\mu$ or NNLO for $ee \to \mu\mu\gamma$)
- resummation using parton shower
- improve / extend NTS (adding to OpenLoops w/ QCD results?) [Laenen et al. 17, 19, 20]

10^{-5} is really small... could we go to $\mathrm{N^3LO}$?

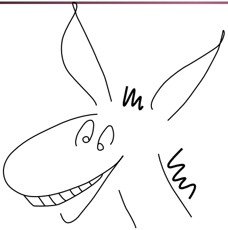
⇒ quite possibly (for the dominant contributions)!

- three-loop heavy-quark form factor: known semi-analytically [Fael, Lange, Schönwald, Steinhauser 22]
- real-virtual-virtual: known for $m_e = 0$ [Gehrmann, Remiddie 01]

⇒ massification to the rescue!

- real-real-virtual: OpenLoops⊕NTS [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini 19; Banerjee, Engel, Schalch, Signer, YU 21; Engel, Signer, YU 21]
- real-real-real: ‘trivial’

kick-off workstop in summer at IPPP!



McMule

P. Banerjee[a], A. Coutinho[b], T. Engel[b,c], A. Gurgone[d,e], F. Hagelstein[f,b], S. Kollatzsch[g], L. Naterop[b,c], M. Rocco[b], N. Schalch[h], V. Sharkovska[b,c], A. Signer[b,c], Y. Ulrich[i]

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[b] Paul Scherrer Institut, CH
[c] University of Zurich, CH
[d] University of Pavia, Italy
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[f] University of Mainz, Germany
[g] TU Dresden, Germany
[h] University of Bern, CH
[i] University of Durham, U.K