

Cavendish HEP Seminars

McMule

QED Corrections for Low-Energy Experiments

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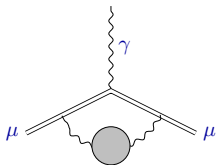
Monte Carlo for MUons and other LEptons

- framework for **fully-differential** (N)NLO in **QED** with $m_f > 0$
- given matrix elements, McMULE takes care of everything else
- subtraction scheme: FKS^ℓ
- quite a few processes already

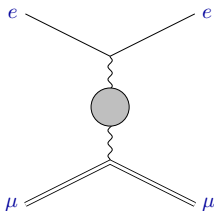
$e\mu \rightarrow e\mu$	NNLO (dominant)	$L \rightarrow \nu\bar{\nu}l$	NNLO
$lp \rightarrow lp$	NNLO (dominant)	$L \rightarrow \nu\bar{\nu}l\gamma$	NLO
$ee \rightarrow ee$	NNLO (ongoing)	$L \rightarrow \nu\bar{\nu}ll^+l^-$	NLO
$ee \rightarrow \gamma\gamma$	NNLO (ongoing)	$L \rightarrow eX$	NLO
$ee \rightarrow \mu\mu$	NNLO (ongoing)		

precision physics at low-energy: $e\mu \rightarrow e\mu$

To get this to $\mathcal{O}(1\%)$...



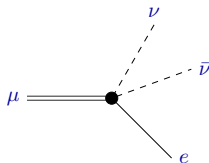
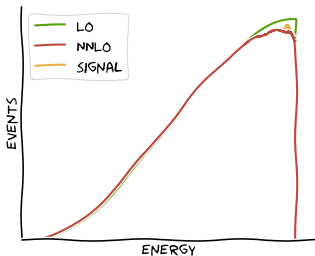
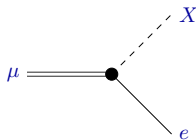
... measure this at 10^{-5}



\Rightarrow NNLO QED with $m_e > 0$ + PS + ...
 (review: [\[MUonE theory initiative 20\]](#))

precision physics at low-energy: $\mu \rightarrow \nu \bar{\nu} e$ and $\mu \rightarrow e X$

- new light cLFV ALP with $m_X \ll m_\mu$
[Calibbi, Redigolo, Ziegler, Zupan 20]
- potentially visible at MEG
[Banerjee, Engel, Gurgone, Papa, Schwendimann, Signer, YU 2?]
- need $\mu \rightarrow \nu \bar{\nu} e$ really precisely



two-loop (if unavailable)

- $\mathcal{M}_n^{(2)}(m=0)$ known [Mastrolia, Prima, et al.] (full $e\mu \rightarrow e\mu$)
- **massify** $\mathcal{M}_n^{(2)}(m) = \mathcal{M}_n^{(2)}(z) + \mathcal{O}(z)$

one-loop and tree-level

- $\mathcal{M}_{n+1}^{(1)}(m)$ and $\mathcal{M}_{n+2}^{(0)}(m)$ using OpenLoops [Pozzorini, Zoller, Lindert et al. 19]
- or by hand using Package-X [Patel 15] / FDF [Fazio, Mastrolia, Marabella, Torres 14]

phase space

- **subtraction scheme** FKS^ℓ [Engel, Signer YU 19]
- numerical integration vegas [Lepage 80]

massification /'masɪ'fɪkeɪʃ(ə)n/

(noun)

Methods to add leading mass effects
to amplitudes in perturbation theory

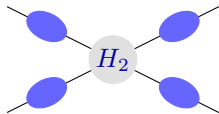
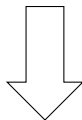
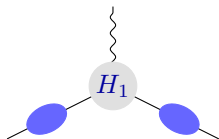
Derivatives:

- **massify** verb
- **massified** adjective

- SCET inspired \sim fragmentation fct.
- drop polynomially suppressed terms

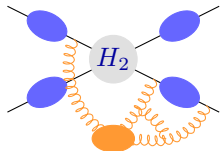
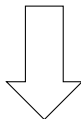
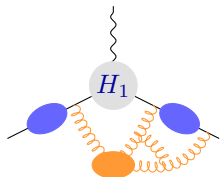
one external mass $m \ll Q^2 = s$

- Bhabha scattering (photonic) [Penin 2005]
- QCD with $n_f = n_m = n_h = 0$ [Mitov, Moch 2006]
- $F_{\gamma^*}(m) \stackrel{!}{\simeq} \sqrt{Z_J} \times \sqrt{Z_J} \times F(0)$
- $Z_J \supset \ln(m^2/\mu^2)$ but **universal**
- $\mathcal{A}_{ee \rightarrow ee}(m)|_{C_F^2} \simeq Z_J^{4/2} \times \mathcal{A}(0)$



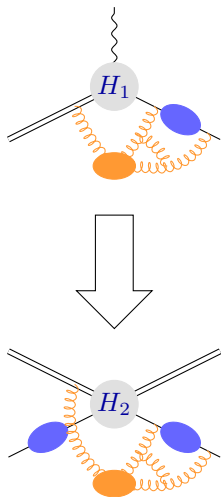
light fermion loops

- n_m terms [Becher, Melnikov 07]
- $F_{\gamma^*}(m) \stackrel{!}{\simeq} \mathcal{S} \times \sqrt{Z_J} \times \sqrt{Z_J} \times F(0)$
- $\mathcal{S}(s, m)$: from vacuum polarisation with massive fermions, $\supset \ln(m^2/s)$
- $\mathcal{A}_{ee \rightarrow ee}(m) \simeq \mathcal{S}' \times Z_J^{4/2} \times \mathcal{A}(0)$



including heavy fermions

- two masses $s \sim M \gg m \gg 0$
[Engel, Gwendiger, Signer, YU 18]
- $F_\mu(m) \stackrel{!}{\simeq} \mathcal{S} \times \sqrt{Z_q} \times F_\mu(0)$
- $\mathcal{A}_{\mu e}(m) \simeq \mathcal{S}' \times Z_q^{2/2} \times \mathcal{A}(0)$



- soft region using light-cone coordinates for q

$$\propto \int d^d k \frac{\Pi^{(n_m)}(k)}{(k^2)^2 (2p \cdot k) (2q_- \cdot k)}$$

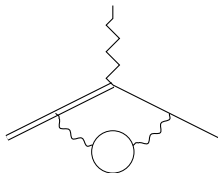
$$\propto \int_0^\infty dx \frac{m^{2-4\epsilon} \Gamma(\epsilon) \Gamma(1-\epsilon)}{x(s + M^2 x)} = \infty$$

- integral is not regularised in d dimensions

\Rightarrow analytic regularisation: $\frac{1}{p \cdot k} \rightarrow \frac{1}{(p \cdot k)^{1+\eta}}$
 [Smirnov 97, Becher, Broggio, Ferroglia 14]

$$\propto \int dx \frac{m^{2-4\epsilon-\eta} \Gamma(\epsilon + \frac{\eta}{2})}{x^{1-\eta/2} (s + M^2 x)^{1+\eta/2}}$$

$$\propto (mM)^{-\eta} \Gamma(\epsilon + \frac{\eta}{2}) \Gamma(\frac{\eta}{2}) \propto \frac{(mM)^{-\eta}}{\eta}$$



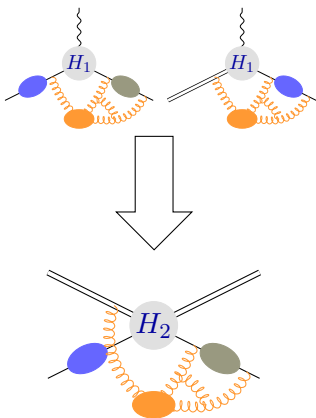
- $Z_q \supset -\frac{s^\eta}{\eta} \rightarrow Z_q \times \mathcal{S} \supset \log \frac{s}{mM}$

⇒ breakdown of naive factorisation \sim factorisation anomaly [Beneke 05, Becher, Bell, Neubert 11]

- new feature for $0 \ll m \ll M$

⇒ two matching calculations

- now $-\frac{s^\eta}{\eta} \subset Z_q \neq \bar{Z}_q \supset \frac{m^{-2\eta}}{\eta}$



0. (anti-)collinear contributions $\sqrt{Z_q}$ and $\sqrt{\bar{Z}_q}$ known
1. calculate $\mathcal{M}_n^{(2)}(0)$ aka. the difficult bit
2. calculate $\mathcal{O}(\epsilon^2)$ of $\mathcal{M}_n^{(1)}(0)$ (you probably already have that)
3. process dependent soft function \mathcal{S} in eikonal theory **with** analytic regulator
4. $\mathcal{M}_n^{(2)}(m) = \prod_i \sqrt{Z_i} \times \prod_i \sqrt{\bar{Z}_i} \times \mathcal{S} \times \mathcal{M}_n^{(2)}(0) + \mathcal{O}(z)$
- n.* resum if needed

FKS^l

or tricking YFS into a full-fledged subtraction scheme

where we stand

- process with extra photon, let $\xi \propto E_\gamma$
- no collinear singularity
- $\mathcal{M}_{n+1}^{(0)} \propto 1/\xi^2$ is divergent
- phase space $d\phi_1 = d\Upsilon d\xi \xi^{1-2\epsilon}$

credo: keep splitting until vegas can integrate it

$$\begin{aligned}
 d\sigma &= d\Phi_n d\phi_1 \mathcal{M}_{n+1}^{(0)} = d\Phi_n d\Upsilon d\xi \xi^{-1-2\epsilon} \underbrace{\left(\xi^2 \mathcal{M}_{n+1}^{(0)} \right)}_{\rightarrow \text{finite}} \\
 &= d\Phi_n d\Upsilon d\xi \left(\underbrace{-\frac{\xi_c^{-2\epsilon}}{2\epsilon} \delta(\xi)}_{\supset d\sigma^{(s)}} + \underbrace{\left(\frac{1}{\xi^{1+2\epsilon}} \right)_c}_{\supset d\sigma^{(h)}} \right) \left(\xi^2 \mathcal{M}_{n+1}^{(0)} \right)
 \end{aligned}$$

- defined c -distribution $\int d\xi \left(\frac{1}{\xi} \right)_c f(\xi) = \int d\xi \frac{f(\xi) - f(0)\theta(\xi_c - \xi)}{\xi}$

- $d\sigma^{(s)}$ easy because $\delta(\xi) \rightarrow$ eikonal $\hat{\mathcal{E}}$
- $d\sigma^{(h)}$ is finite $\epsilon \rightarrow 0$

$$d\sigma^{(s)} \subset d\sigma_n(\xi_c) = d\Phi_n \left(\mathcal{M}_n^{(1)} + \hat{\mathcal{E}}(\xi_c) \mathcal{M}_n^{(0)} \right)$$

$$d\sigma^{(h)} \equiv d\sigma_{n+1}(\xi_c) = d\Phi_{n+1} \left(\frac{1}{\xi} \right)_c \left(\xi^2 \mathcal{M}_{n+1}^{(0)} \right)$$

ξ_c is unphysical

- both bits depend on ξ_c but σ doesn't
- \Rightarrow dependence drops out **exactly**
- use to test implementation and stability

credo: keep splitting until vegas can integrate it

- no one-loop eikonals \rightarrow like normal FKS
- $d\sigma^{(h)}$ is not yet finite ($\mathcal{M}_{n+1}^{(1)}$ has $1/\epsilon$ poles)
- use KLN theorem / YFS for $(n+1)$ -particle process

$$\underbrace{\mathcal{M}_{n+1}^{(1)f}}_{\supset d\sigma^{(fin)}} = \mathcal{M}_{n+1}^{(1)} + \underbrace{\hat{\mathcal{E}}(\xi_c)\mathcal{M}_{n+1}^{(0)}}_{\supset -\mathcal{I}} = \text{finite}$$

\Rightarrow eikonal subtraction

- $d\sigma^{(h)}(\xi_c) = d\sigma^{(fin)}(\xi_c) - \mathcal{I}(\xi_c)$
- \mathcal{I} is process- and observable dependent, not finite
- deal with it later

credo: keep splitting until vegas can integrate it

- two soft photons → just do FKS twice

$$d\sigma = d\sigma^{(hh)} + 2 \times d\sigma^{(hs/sh)} + d\sigma^{(ss)}$$

- $d\sigma^{(ss)} \propto \frac{1}{2!} \hat{\mathcal{E}}^2(\xi_c) \mathcal{M}_n^{(0)}$
- $d\sigma^{(hh)} = \frac{1}{2!} d\Phi_{n+2} \left(\frac{1}{\xi_1} \right)_c \left(\frac{1}{\xi_2} \right)_c \left(\xi_1^2 \xi_2^2 \mathcal{M}_{n+2}^{(0)} \right)$
- $d\sigma^{(hs/sh)} = 2 \times \frac{1}{2!} d\Upsilon d\xi \left(\frac{1}{\xi^{1+2\epsilon}} \right)_c \left(\xi^2 \hat{\mathcal{E}}(\xi_c) \mathcal{M}_{n+1}^{(0)} \right) = \mathcal{I}(\xi_c)$

everything complicated drops out!

- adding two-loop $\mathcal{M}_n^{(2)}$

$$d\sigma_n(\xi_c) = d\Phi_n \underbrace{\left(\mathcal{M}_n^{(2)} + \hat{\mathcal{E}} \mathcal{M}_n^{(1)} + \frac{1}{2!} \mathcal{M}_n^{(0)} \hat{\mathcal{E}}^2 \right)}_{\mathcal{M}_n^{(2)f}}$$

$$d\sigma_{n+1}(\xi_c) = d\Phi_{n+1} \left(\frac{1}{\xi_1} \right)_c \left(\xi_1^2 \mathcal{M}_{n+1}^{(1)f} \right)$$

$$d\sigma_{n+2}(\xi_c) = d\Phi_{n+2} \left(\frac{1}{\xi_1} \right)_c \left(\frac{1}{\xi_2} \right)_c \left(\xi_1^2 \xi_2^2 \mathcal{M}_{n+2}^{(0)} \right)$$

- never need $\mathcal{M}_{n+i}^{(j)}$ higher than $\mathcal{O}(\epsilon^0)$
- YFS build-up
- can be extended to N^ℓLO \Rightarrow FKS^ℓ



McMULE

putting everything together

- public repository

<https://gitlab.com/mule-tools/mcmule/>

- core written in Fortran 95, helper scripts in python
 - 7 kLoC (core) + 630 kLoC (matrix elements) + 5 kLoC (helper)
 - use COLLIER for some loop integrals [Denner, Dittmaier, Hofer 16]
 - adding new processes is simple
 - pre-compiled Docker container available
 - all published data available online
 - ease of use (both for theory and experimental)
- user needs to only touch one file specifying observable
- arbitrary cuts and 1D histograms

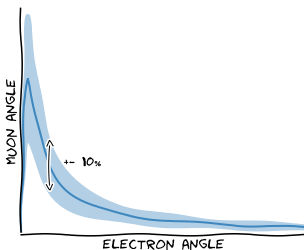
$e\mu \rightarrow e\mu$ for the MUonE experiment @ CERN's M2

- 150 GeV μ^\pm , atomic e^-
- \Rightarrow everything in the labframe
- can't measure energies well
- \Rightarrow only got $\theta_{e,\mu}$ and $\varphi_{e,\mu}$
- LO kinematics $\theta_\mu = \theta_\mu(\theta_e)$
- only dominant NNLO

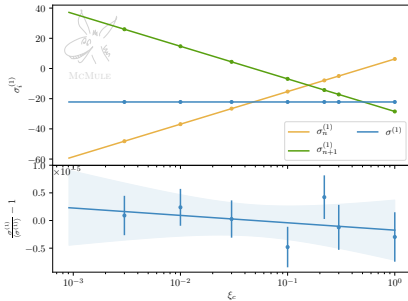


cuts

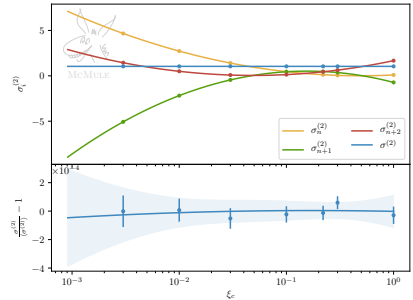
- $E_e > 1 \text{ GeV}$, $\theta_\mu > 0.3 \text{ mrad}$
- $b = \left| \frac{\theta_\mu}{\theta_e} - 1 \right| < 10\%$
(optional)



- slightly different observable

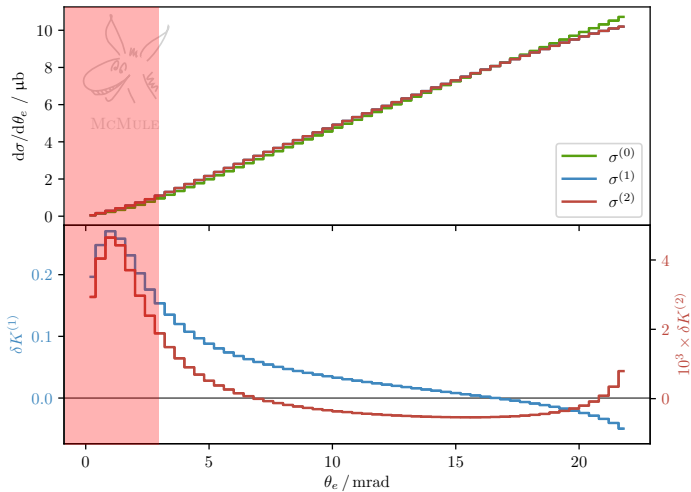


verified with [Alicevich, Carloni Calame, Chiesa, Montagna, Nicosini, Piccinini 19]

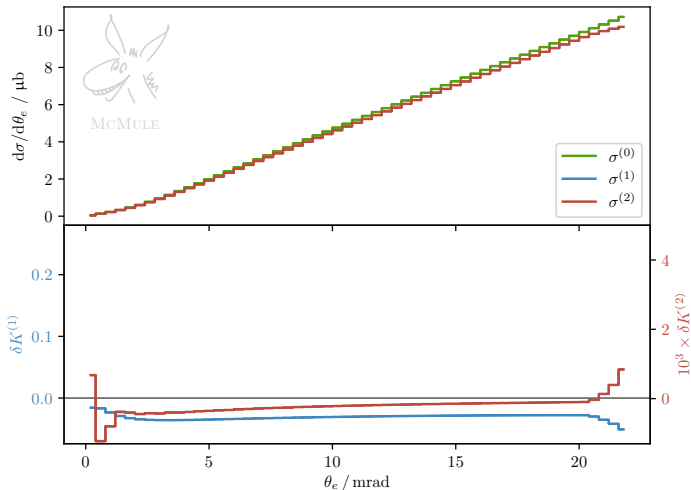


verified with [Carloni Calame, Chiesa, Hasan, Montagna, Nicosini, Piccinini 20]

⇒ need to resum

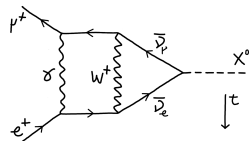


- corrections are smaller and flatter

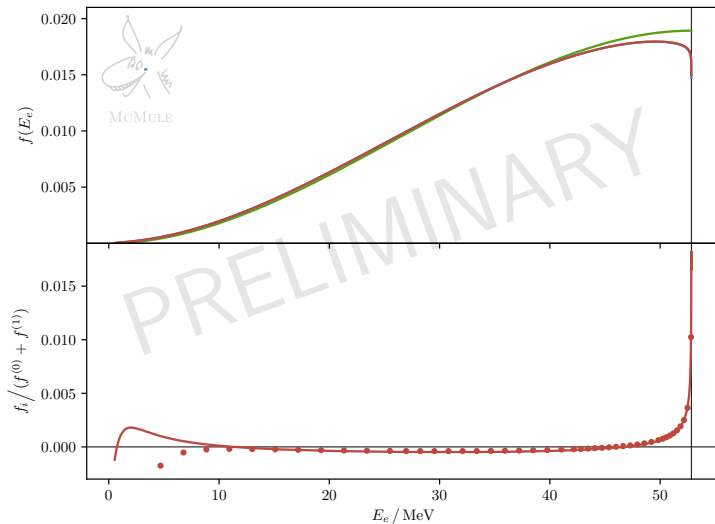
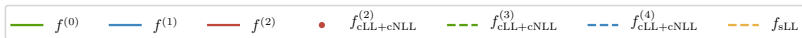


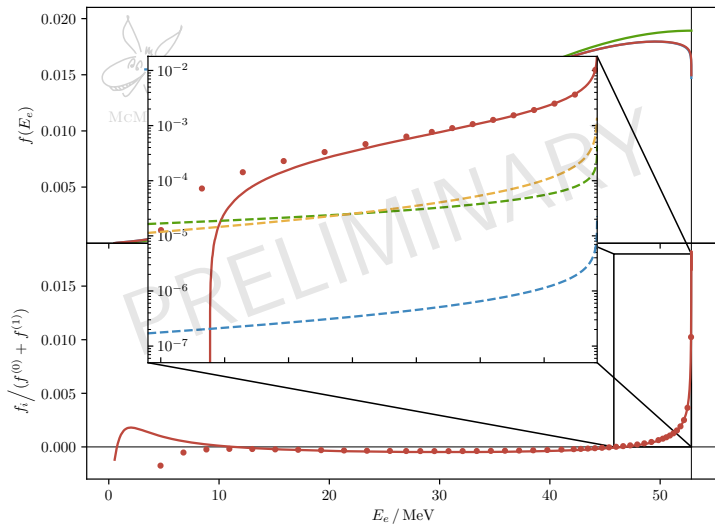
joint project with the MEG collaboration [Banerjee, Engel, Gurgone, Papa, Schwendimann, Signer, YU 2?]

- search for cLFV ALP X
- looking for small bumps in muon decay spectrum
- full detector simulation interfaced to MCMULE
- $$\frac{d^2\Gamma}{dE_e d\cos\theta} = \Gamma_0 \left(F(E_e) + G(E_e)P \cos\theta \right)$$
- need best possible F and G (NNLO + sLL + ...)
- signal at NLO

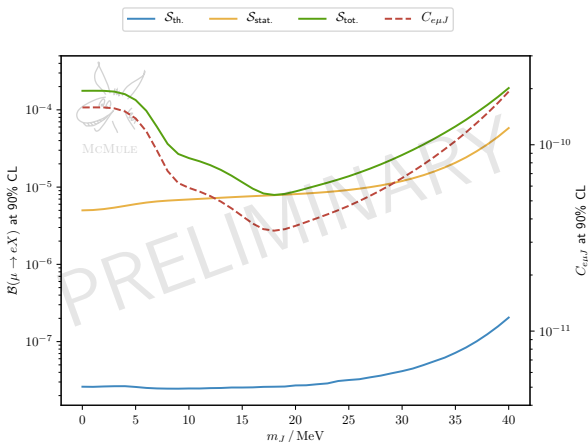


[Gurgone 20]





- combine theory with simplified (but vetted) detector model
- realistic $N = 10^8 \mu$ (due to trigger)
- analysis for dedicated search ongoing



conclusion

- low-energy QED is very important for many experiments
- MCMULE is a framework to implement these
- used in experimental analysis (MEG, MUonE)

future, in somewhat order

- upcoming processes $ee \rightarrow ee$ (\rightarrow PRad II), $ee \rightarrow \gamma\gamma$ (\rightarrow PADME), $ee \rightarrow \mu\mu$ (\rightarrow Belle), full $e\mu \rightarrow e\mu$ at NNLO
- adding parton shower
- improve stability as $\xi \rightarrow 0$ in $\mathcal{M}_{n+1}^{(1)} \rightarrow$ next-to-eikonal
- 2γ exchange in $\ell p \rightarrow \ell p$ (\rightarrow MUSE, PRad, ..)
- use FKS³ for dominant N³LO in $e\mu \rightarrow e\mu$



MCMULE

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