

PSI 2019

High-precision QED predictions for low-energy lepton experiments

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what this talk is not about

- bound-state QED
- hadronic effects

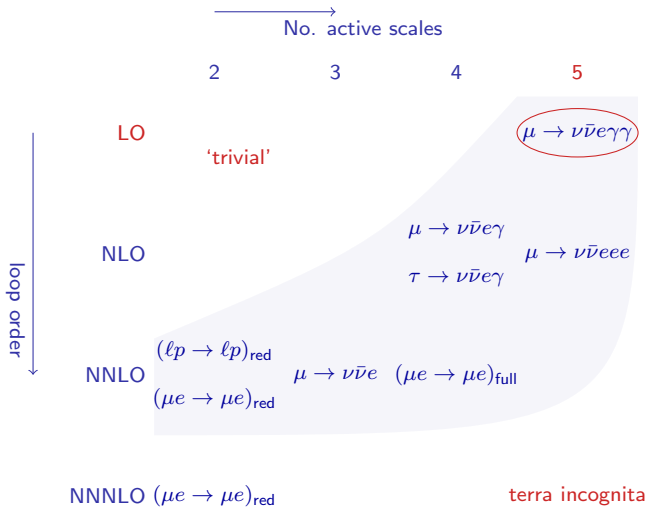
what this talk is about

- tailored predictions
- in perturbative QED
- of background studies
- for lepton experiments (MEG, Mu3e, MUSE, MUonE etc)

take LHC techniques and apply to lepton experiments

LHC techniques (N)NLO, (N)NLL, NLO+PS...

- higher-order technology is mature
- $\alpha_s(Q_{\text{LHC}}) \gg \alpha_{\text{em}} \sim 1\% \rightarrow$ much higher precision
- **but** large logarithms like $\alpha_{\text{em}} \log \frac{m_e^2}{m_\mu^2} \sim 10\%$
- no PDF \rightarrow less hadronic problems
- people involved $\mathcal{O}(10^3)$ v. $\mathcal{O}(10)$
- very few examples at NNLO done: Bhabha scattering and muon decay
- more required: $\mu e \rightarrow \mu e$, $lp \rightarrow lp$, $ee \rightarrow ee$, $ee \rightarrow \gamma\gamma$, ...

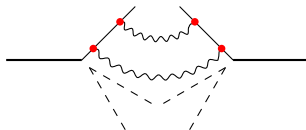


background to $\mu \rightarrow eX, X \rightarrow \gamma\gamma$ @ MEG

phase space

matrix element

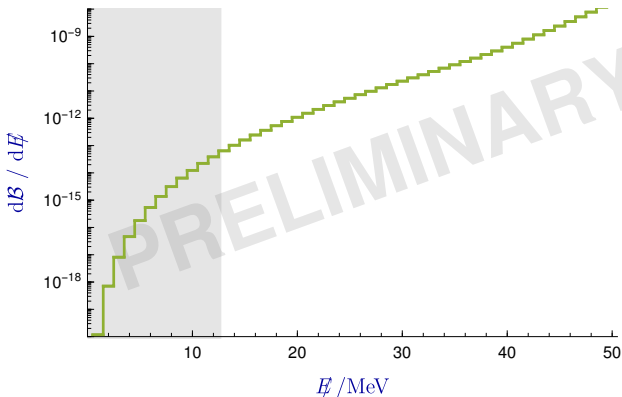
$$\int_{\text{MEG}} d\Phi_5$$



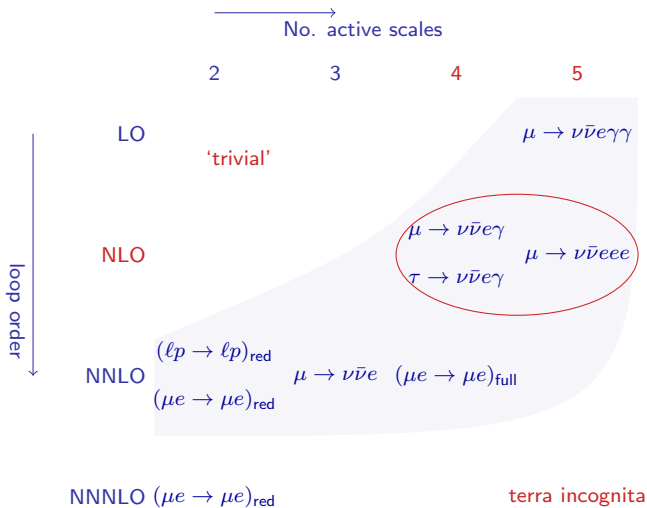
numerical

trivial, analytic

MEG I cuts for photon $\angle(\gamma_1, \gamma_2) > 16.4^\circ$ at LO



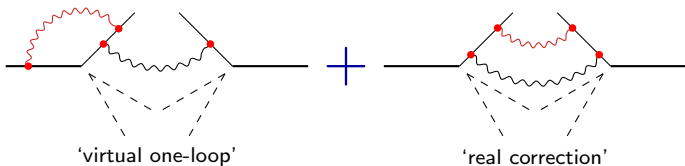
less than $\mathcal{B} < 10^{-13}$ in the shaded region



why higher-order?

- reliable background studies (e.g. $E_T \rightarrow 0$, angular distr.)
- fiducial measurement \rightarrow PDG value (e.g. τ decay)

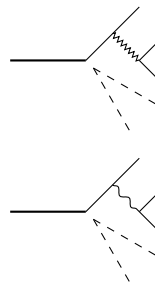
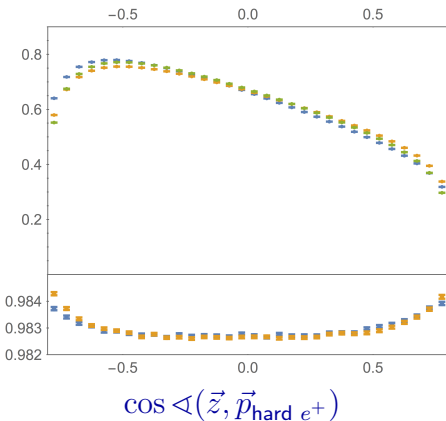
how higher-order?



- virtual and real correction separately divergent
- from LHC pheno: dimensional regularisation (no m_γ etc.)
- we use FKS subtraction [Frixione, Kunstz, Signer 1995]

light new physics would change angular distributions

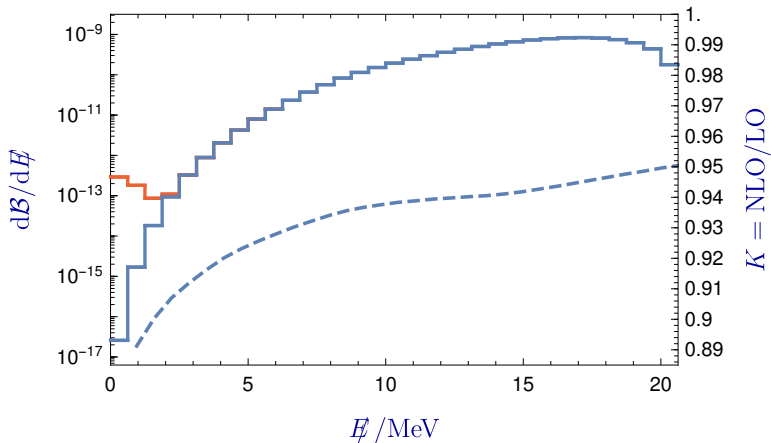
$$K = \frac{N_{LO}}{LO} \frac{dB}{d \cos \theta}$$



[cf. poster #73
by N. Berger]

[Pruna, Signer, YU 2016]

exactly one photon $E_\gamma > 40\text{MeV}$ in the detector. $\mathcal{B}_{\text{NP}} \simeq 4.2 \cdot 10^{-13}$



- large K (dashed) \Rightarrow large logs $\sim \log \frac{E_{\text{cut}}^2}{M^2}$

[Pruna, Signer, YU 2017]

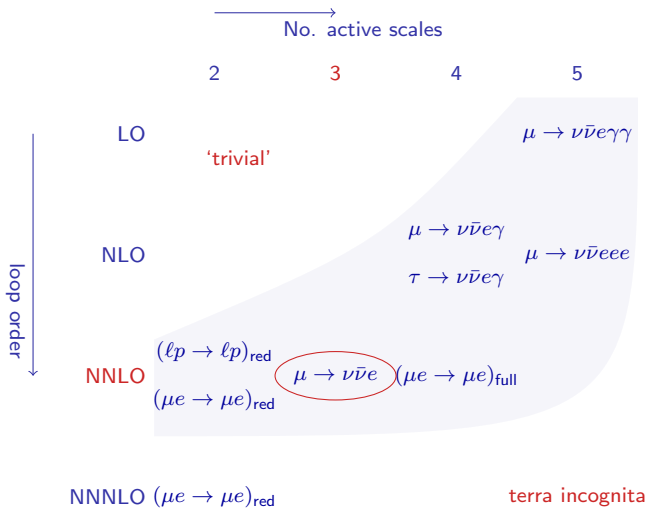
radiative τ decay

	$100 \times \mathcal{B}$	
BABAR	1.847(54)	[Oberhof et al 2015]
LO	1.834	0.2σ
NLO	1.645(1)	3.7σ [Fael, Mercolli, Passera 2015]
$\epsilon_{\text{HO}} \times \text{BABAR}$	1.740(50)	1.4σ

- BABAR uses very restrictive cuts, unfolds using LO MC
- implement cuts at NLO and LO and compare

$$\epsilon_{\text{HO}} = \frac{\epsilon_{\text{NLO}}}{\epsilon_{\text{LO}}} = \frac{\mathcal{B}_{\text{NLO}}(10 \text{ MeV})}{\mathcal{B}_{\text{NLO}}(\text{BABAR})} \frac{\mathcal{B}_{\text{LO}}(\text{BABAR})}{\mathcal{B}_{\text{LO}}(10 \text{ MeV})} = 0.92$$

- large logs $\sim \log \frac{E_{\text{cut}}^2}{M^2}$



(see also poster #72 by Tim Engel)

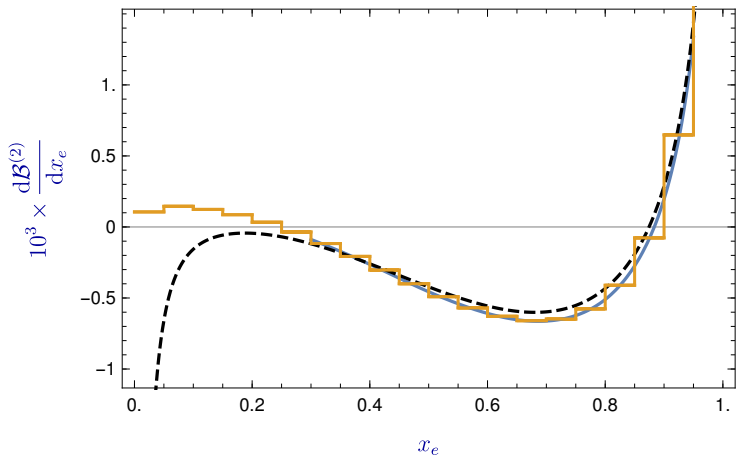
muon decay at NNLO: history calculations

- inclusive NNLO for G_F [Stuart, van Ritbergen 1999]
- logarithms $\log^{\{1,2\}} \frac{m_e^2}{m_\mu^2}$ of $d\Gamma/dE_e$
[Arbuzov, Czarnecki, Gaponenko 2002, Arbuzov, Melnikov 2002]
- fully inclusive, numeric energy spectrum [Anastasiou, Melnikov, Petriello 2005]

how?

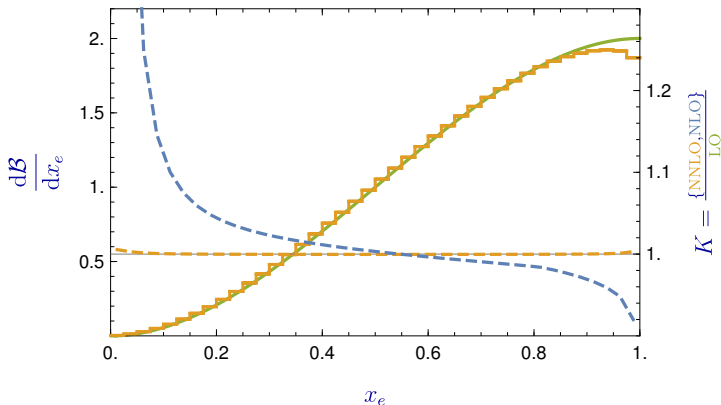
- analytic two-loop integrals [Chen 2018] and form factors
[Engel, Gnendiger, Signer, YU 2018]
- fully differential Monte Carlo [Engel, Signer YU, 2019] using
FKS²

electron energy spectrum

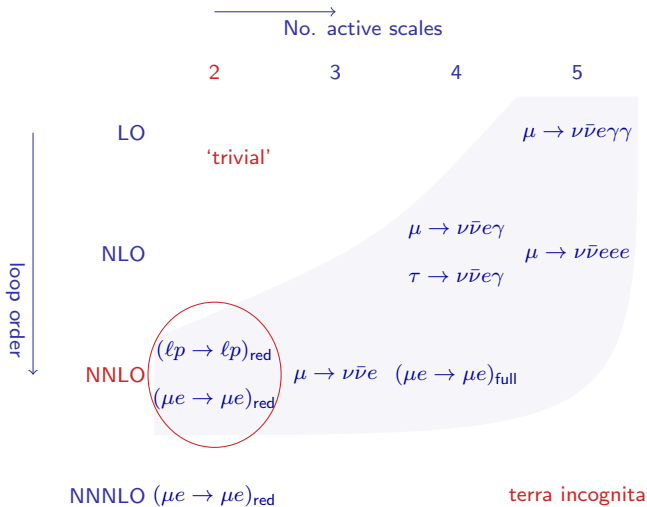


our result, Anastasiou et al, logarithms

total photon energy within $\cos \angle(\vec{p}_e, p_\gamma) > 0.8$ is $\sum E_\gamma < 10\text{MeV}$



NNLO K -factor **dashed**, large logs in tail \Rightarrow resummation

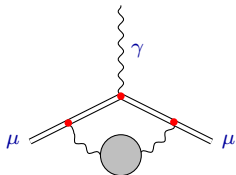


(see also poster #72 by Tim Engel)

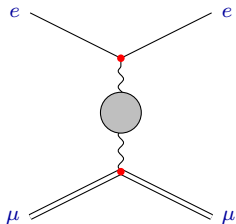
- measure HVP using t -channel μ - e scattering at 10ppm

$$a_{\mu}^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \underbrace{\Delta\alpha_{\text{had}}\left(\frac{x^2}{x-1} m_{\mu}^2\right)}_{\propto d\sigma/dt}$$

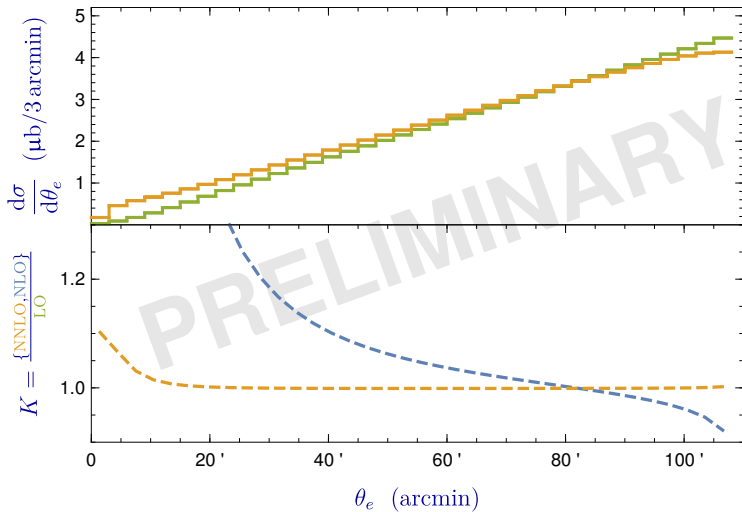
[Carloni Calame et al. 2015, Abbiendi et al 2016]



- proposed experiment @ CERN's M2
 [Matteuzzi, YU et al 2019 (LOI)]
- huge theory effort [Alacevich et al 2018, Mastrolia et al 2017 and 2018, Fael, Passera 2019]



⇒ NNLO emission-from- e -line-only (largest part) [Banerjee, Engel, Signer, YU soon] using [Bernreuther et al. 2004] with FKS²

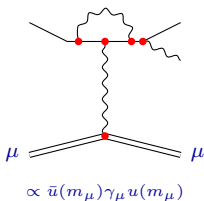


- extract form factors at high precision at low Q^2
(MESA, MUSE, Prad, QWeak, ...)

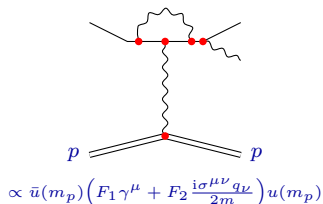
[cf. talks by Golossanov (#141) and Antognini (#63)]

⇒ QED corrections (cf. at NLO [Gramolin 2014, Akushevich et al. 2015] and NNLO [Bucoveanu, Spiesberger 2018])

- emission-from- ℓ -line-only relatively straight-forward

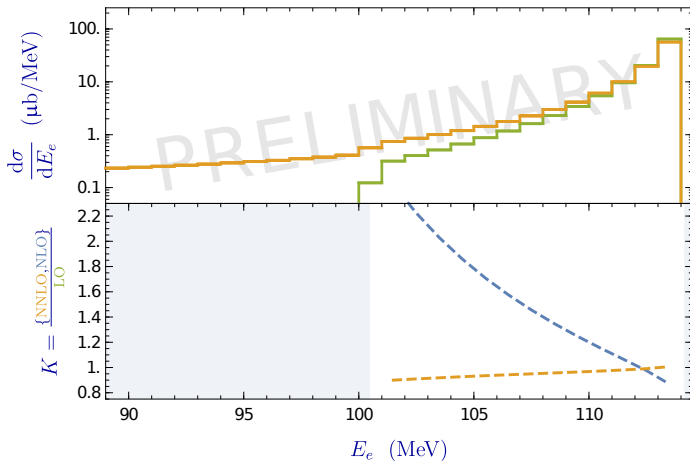


$$\propto \bar{u}(m_\mu) \gamma_\mu u(m_\mu)$$

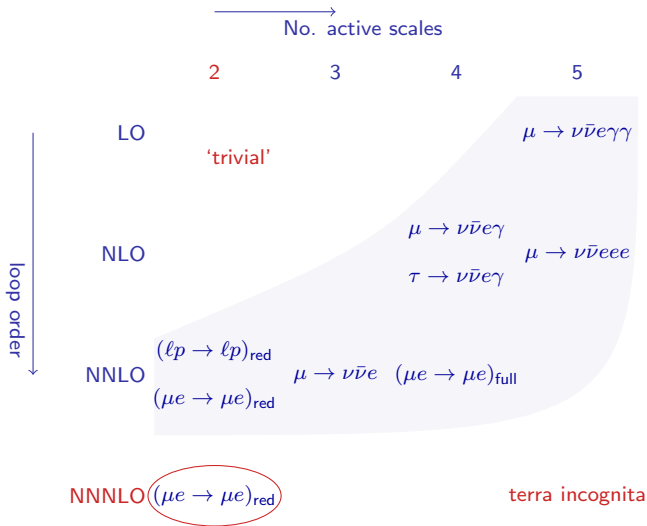


$$\propto \bar{u}(m_p) \left(F_1 \gamma^\mu + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{2m} \right) u(m_p)$$

MUSE cuts $20^\circ < \theta < 100^\circ$, $p_{\text{in}} = 115$ MeV



- shaded region kinematically forbidden at tree-level



for emission-from- e -line-only in μ - e scattering

- extension FKS^ℓ is known
 - three-loop $\gamma^* \rightarrow Q\bar{Q}$ being calculated [Henn, Smirnov, Smirnov, Steinhauser 2016, Lee, Smirnov, Smirnov, Steinhauser 2018, Ablinger, Blümlein, Marquard, Rana, Schneider 2018...]
 - adapt two-loop $\gamma^* \rightarrow q\bar{q}g$ [Garland, Gehrmann, Glover, Koukoutsakis, Remiddi 2002]
- ⇒ no conceptual problems for N^3LO
- but **many** practical problems

conclusion

- NNLO (and beyond) achievable for many processes
- numerical instabilities along the way

outlook

- resummation for tails \Rightarrow integrate YFS PS !
- add $\ell p \rightarrow \ell p$, $e\mu \rightarrow e\mu$, $\mu \rightarrow eJ$, $ee \rightarrow ee$, $ee \rightarrow \gamma\gamma$...
- bundle all of this into one tool

...introducing MCMULE

(Monte Carlo for Muons and other leptons)

P. Banerjee, T. Engel, A. Signer, YU, ...