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Masses in QED calculations

Massification and subtraction

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why massive QED?

technical reasons

- simplifies real corrections
- initial state logarithms $\log m^2/\{s,t,u\}$ explicit

phenomenological reasons

- can consider more exclusive variables (eg. energy spectrum)
- $L = \log \frac{m_e^2}{m_\mu^2} \sim -10$
- $\mu - e$ scattering at 10ppm: NNLO $\alpha^2 L^2 \gg 10^{-5}$

processes

μ decay

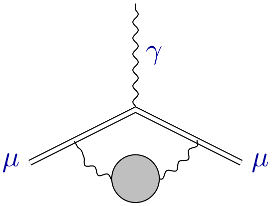
- NNLO calculation using optical theorem [Ritbergen, Stuart 99]
- leading logs of energy spectrum [Arbuzov, Czarnecki, Gaponenko; Arbuzov, Melnikov 02]
- $f(E_e)$ numerically with full m_e [Anastasiou, Melnikov, Petriello 05]
- $m_e = 0$ form factors [Bell 07, Bonciani, Ferroglia 08]
- $m_e > 0$ master integrals [Chen 18]

$\mu - e$ scattering

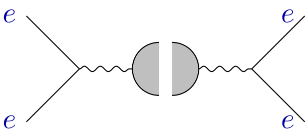
- NLO with $m_e = 0$ [Nikishov 61, Eriksson 61, ...]
- NLO with $m_e > 0$ + EW [Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini 18]
- NNLO master integrals with $m_e = 0$ [Mastrolia, Passera, Primo, Schubert 17, Di Vita, Laporta, Mastrolia, Primo, Schubert 18]

muon $g - 2$

How to get this to $< 1\%$



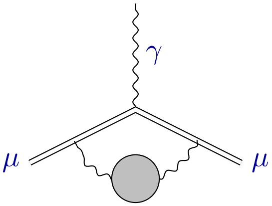
Traditional: time-like data



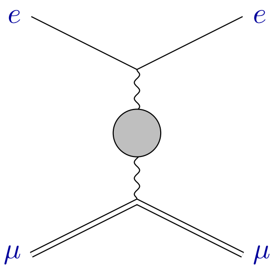
[Davier 07, Jegerlehner, Nyffeler 09, Teuber 11,...]

A novel approach: space-like

To get this to $\mathcal{O}(1\%)$ ...



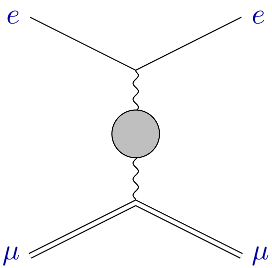
... measure this at 10^{-5}



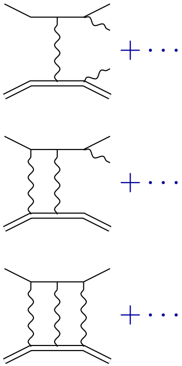
[Carlo Calame, Passera, Trentadue, Venanzoni 15, G. Abbiendi et al 17]

$\mu - e$ scattering

To measure this at 10^{-5} ...



... calculate this at NNLO



strategy

goal: want massive-ish calculation of QED processes

- calculate $\mathcal{M}_n^{(2)}(0)$ [Bonciani, Ferroglia 08] (μ decay)
  [Mastrolia, Primo, Torres Bobadilla et al] ($\mu - e$)
- massify $\mathcal{M}_n^{(2)}(m) = \mathcal{M}_n^{(2)}(z) + \mathcal{O}(z)$
- calculate $\mathcal{M}_{n+1}^{(1)}(m)$ and $\mathcal{M}_{n+2}^{(0)}(m)$ [Fael, Mercolli, Passera 15, Pruna, Signer, YU 17] (μ decay)
- integrate PS with novel subtraction scheme FKS^2
- result is correct up to $\mathcal{O}(z)$

massification /ˈmasɪˈfɪkeɪʃ(ə)n/

(noun)

Methods to add leading mass effects to amplitudes in perturbation theory

Derivatives:

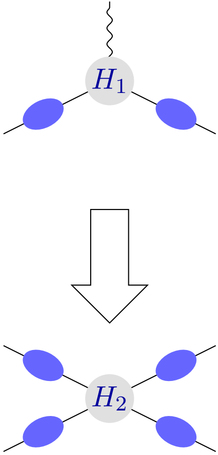
- **massify** verb
- **massified** adjective

Villigen English Dictionary (2019)

massification: brief history C_F

one external mass $m \ll Q^2 = s$

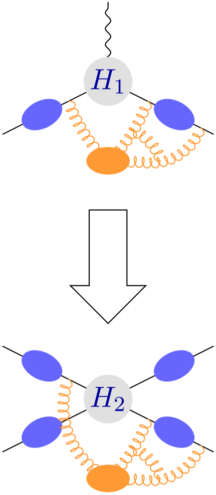
- SCET inspired $\sim$ fragmentation fct.
- Bhabha scattering (photonic) [Penin 2005]
  matching $1/\epsilon \to \ln m_e, \ln m_\gamma$
- QCD with $n_f = n_m = n_h = 0$ [Mitov, Moch 2006]
- matching
  $F_{\gamma^*}(m) = \sqrt{Z_J} \times \sqrt{Z_J} \times F(0)$
- Z_J: jet fct., independent of hard scale s, $\supset \ln(m^2/\mu^2)$



massification: brief history n_m

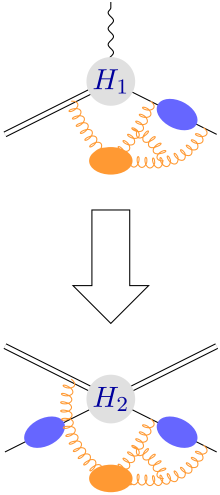
light fermion loops

- n_m terms [Becher, Melnikov 07]
- $F_{\gamma^*}(m) = \mathcal{S}\times\sqrt{Z_J}\times\sqrt{Z_J}\times F(0)$
- $\mathcal{S}(s,m)$: soft function, only contributions from vacuum polarization diagrams with massive fermions, $\supset \ln(m^2/s)$
- $\mathcal{A}_{ee\to ee}(m) = \mathcal{S}' \times Z_J^{4/2} \times \mathcal{A}(0)$
- factorisation $\leftrightarrow$ resummation via RG equations



massification: brief history n_h

- two different masses $M \gg m \gg 0$ [Engel, Gnendiger, Signer, YU 18]
- $F_\mu(m) = \mathcal{S} \times \sqrt{Z_q} \times F_\mu(0)$
- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times \sqrt{Z_q \times \bar{Z}_q} \times \mathcal{A}(0)$



$m = 0 \;\to\; m \neq 0$

- extend this to processes with two external masses, M and m (in QCD)
- calculate simplest example $t(p) \to b(q) + W^\pm$ and compare with $\gamma^* \to q\bar{q}$ (replace t with μ etc)
- identify momentum regions h, c, s and us (drops out)
- reduction mixes soft/collinear regions confusingly $\Rightarrow$ do this at the diagram level

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example: n_m contribution

- soft region

$$\propto \int \mathrm{d}^d k \frac{\Pi^{(n_m)}(k)}{(k^2)^2(2p\cdot k)(2q_-\cdot k)}$$

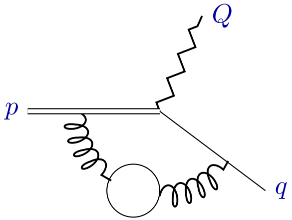
$$\propto \int_0^\infty \mathrm{d}x \frac{m^{2-4\epsilon}\Gamma(\epsilon)\Gamma(1-\epsilon)}{x(s+M^2x)} = \infty$$

- integral is not regularised in d dimensions

$\Rightarrow$ analytic regularisation: $\frac{1}{p\cdot k} \to \frac{1}{(p\cdot k)^{1+\eta}}$ [Smirnov 97, Becher, Broggio, Ferroglia 14]

$$\propto \int \mathrm{d}x \frac{m^{2-4\epsilon-\eta}\Gamma(\epsilon+\frac{\eta}{2})}{x^{1-\eta/2}(s+M^2x)^{1+\eta/2}}$$

$$\propto (mM)^{-\eta}\Gamma(\epsilon+\tfrac{\eta}{2})\Gamma(\tfrac{\eta}{2}) \propto \frac{(mM)^{-\eta}}{\eta}$$

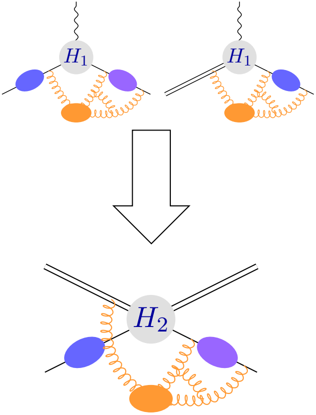


the anomaly

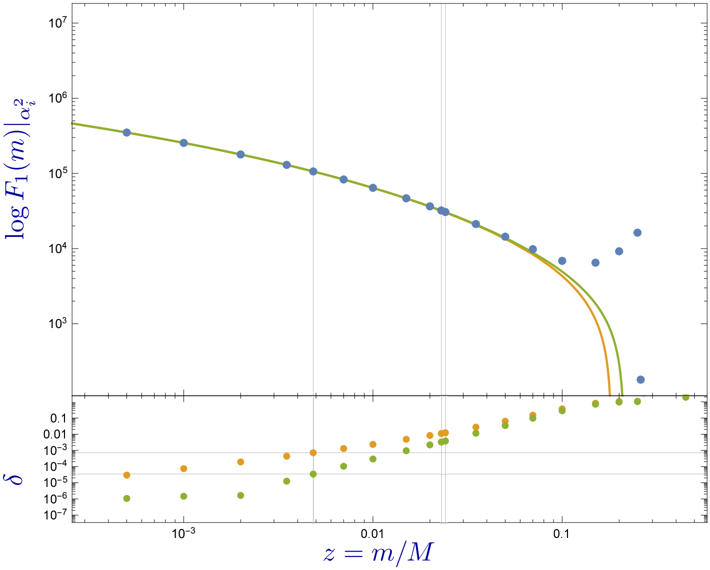
- note: always expand η before ϵ!
- $Z_q \supset -\frac{s^\eta}{\eta} \rightarrow \; Z_q \times \mathcal{S} \supset \log \frac{s}{mM}$

$\Rightarrow$ breakdown of naive factorisation $\sim$ factorisation anomaly [Beneke 05, Becher, Bell, Neubert 11]

- new feature for $0 \ll m \ll M$
- now $-\frac{s^\eta}{\eta} \subset Z_q \neq \bar{Z}_q \supset \frac{m^{-2\eta}}{\eta}$



$\log F_1$



10^7
10^6
10^5
10^4
10^3
$\log F_1(m)|_{\alpha_i^2}$
0.1
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
δ
10^{-3}
0.01
0.1
$z = m/M$

recipe

0. (anti-)collinear contributions $\sqrt{Z_q}$ and $\sqrt{\bar{Z}_q}$ known
1. calculate $\mathcal{M}_n^{(2)}(0)$ aka. the difficult bit
2. calculate $\mathcal{O}(\epsilon^2)$ of $\mathcal{M}_n^{(1)}(0)$ (you probably already have that)
3. process dependent soft function $\mathcal{S}$ in eikonal theory with analytic regulator
4. $\mathcal{M}_n^{(2)}(m) = \prod_i \sqrt{Z_i} \times \prod_i \sqrt{\bar{Z}_i} \times \mathcal{S} \times \mathcal{M}_n^{(2)}(0) + \mathcal{O}(z)$

n. resum if needed

FKS^2: double-soft extension of FKS

real correction at NLO

The FKS formalism at NLO [Frixione, Kunszt, Signer 95, Frederix, Frixione, Maltoni, Stelzer 09]

- no collinear singularities in μ–e $\to$ very simple scheme
- everything that vegas sees needs to be finite!
- let $E_\gamma \propto \xi$

$$\underbrace{\mathrm{d}\phi_{n+1}}_{\propto \xi^{1-2\epsilon}\mathrm{d}\xi}\ \underbrace{\mathcal{M}^{(0)}_{n+1}}_{\supset \xi^{-2}} \propto \mathrm{d}\phi_n \times \mathrm{d}\xi\,\mathrm{d}\Omega\ \xi^{-1-2\epsilon}\underbrace{\Big(\xi^2\mathcal{M}^{(0)}_{n+1}\Big)}_{\text{reg. }\xi\to 0}$$

- introduce arbitrary $0 < \xi_\text{cut} \le 1$

$$\propto \mathrm{d}\Omega\left(\underbrace{-\frac{{\xi_\text{cut}}^{-2\epsilon}}{2\epsilon}\delta(\xi)}_{(s)} + \underbrace{\Big(\xi^{-1-2\epsilon}\Big)_{\xi_\text{cut}}}_{(h)}\right)\Big(\xi^2\mathcal{M}^{(0)}_{n+1}\Big)$$

with $\int \mathrm{d}\xi\,(\xi^n)_{\xi_\text{cut}} f(\xi) = \int \mathrm{d}\xi\,\xi^n\Big(f(\xi) - f(0)\theta(\xi_\text{cut}-\xi)\Big)$

real correction at NLO

- $\mathrm{d}\sigma^{(h)}$ is finite $\Rightarrow$ $\epsilon \to 0$ numerically
- $\mathrm{d}\sigma^{(s)}$ is 'trivial' because of $\delta(\xi)$

$$\begin{aligned}\mathrm{d}\sigma^{(s)} &\propto \frac{{\xi_\text{cut}}^{-2\epsilon}}{2\epsilon}\int \mathrm{d}\Omega\, \Big(\xi^2\mathcal{M}_{n+1}^{(0)}\Big)_{\xi=0} \\ &\propto \frac{{\xi_\text{cut}}^{-2\epsilon}}{2\epsilon}\mathcal{M}_n^{(0)}\int \mathrm{d}\Omega\ \mathcal{E} \\ &= \hat{\mathcal{E}}(\xi_\text{cut})\mathcal{M}_n^{(0)}\end{aligned}$$

- eikonal $\mathcal{E}$ is build from building blocks
- $\hat{\mathcal{E}}\mathcal{M}_n^{(0)} + \mathcal{M}_n^{(1)} =$ finite (KLN)
- use ξ_cut to test implementation (any IR safe $\frac{\mathrm{d}^n\sigma}{\mathrm{d}x_1\cdots\mathrm{d}x_n}$ can't depend on ξ_cut)

FKS extension to NNLO: FKS^2

term 1: $\mathcal{M}^{(1)}_{n+1}$ (real × virtual)

- like NLO, but $\mathrm{d}\sigma^{(h)} \propto \mathcal{M}^{(1)}_{n+1}$ is not finite ⇒ split further
  1. chop the pole (and induced terms): $\overline{\text{MS}}$-like subtraction
  2. $\mathcal{M}^{(1)}_{n+1} = \underbrace{\mathcal{M}^{(1)f}_{n+1}}_{\Rightarrow \mathrm{d}\sigma^{(fin)}} \underbrace{-\hat{\mathcal{E}}(\xi^2_{\text{cut}})\mathcal{M}^{(0)}_{n+1}}_{\Rightarrow \mathrm{d}\sigma^{(sin)}}$: eikonal subtraction

     $\mathcal{M}^{(1)f}_{n+1}$ is finite (KLN of the $n+1$ process)
- $\mathrm{d}\sigma^{(fin)}$ and $\mathrm{d}\sigma^{(sin)}$ depend on two a-priori different ξ^i_{cut}
- $\mathrm{d}\sigma^{(fin)}$ is now finite ⇒ $\epsilon \to 0$
- we postpone

$$\mathrm{d}\sigma^{(sin)} \equiv -\mathcal{I}(\xi^1_{\text{cut}}, \xi^2_{\text{cut}}) \propto \int_{\xi^1_{\text{cut}}} \hat{\mathcal{E}}(\xi^2_{\text{cut}})\ (\xi^2 \mathcal{M}^{(0)}_{n+1})$$

FKS extension to NNLO: FKS^2

term 2: $\mathcal{M}^{(0)}_{n+2}$ (real × real)

- do normal FKS twice with two ξ_{cut}, but (h) mixes with (s)

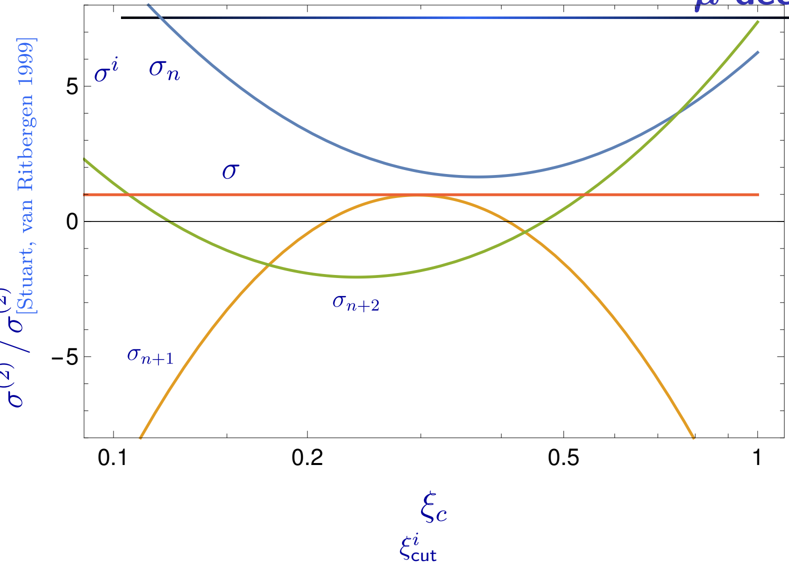
$$\mathrm{d}\sigma_{n+2} = \underbrace{\mathrm{d}\sigma^{(hh)}}_{\epsilon\to 0} + \underbrace{\mathrm{d}\sigma^{(ss)}}_{\hat{\mathcal{E}}(\xi^A_{\text{cut}})\,\hat{\mathcal{E}}(\xi^B_{\text{cut}})} + \mathrm{d}\sigma^{(hs)} + \mathrm{d}\sigma^{(sh)}$$

- mixed terms $\mathrm{d}\sigma^{(hs)}$ are troublesome

$$\mathrm{d}\sigma^{(hs)} \propto \frac{1}{2!}\int_{\xi^A_{\text{cut}}} \hat{\mathcal{E}}(\xi^B_{\text{cut}})\ (\xi^2\mathcal{M}^{(0)}_{n+1}) = \frac{1}{2!}\mathcal{I}(\xi^A_{\text{cut}},\xi^B_{\text{cut}})$$

- luckily $\mathrm{d}\sigma^{(aux)} \equiv \mathrm{d}\sigma^{(sin)} + \mathrm{d}\sigma^{(hs)} + \mathrm{d}\sigma^{(sh)} = 0$ if all ξ^i_{cut} are equal
- the sum $\mathrm{d}\sigma^{(ss)} + \mathrm{d}\sigma^{(s)} + \mathrm{d}\sigma^{(aux)} + \mathrm{d}\sigma_{VV}(m)$ is finite (KLN)

μ decay



$$\sigma_{n+1} + \sigma_{n+2} = \sigma^{hh} + \sigma^{fin} + \sigma^{ss} + \sigma^{s}$$

summary

what we have done

- generalised massification to two external masses
- (some) integrals are not regularised in DREG
- factorisation anomaly breaks naive factorisation
- novel subtraction scheme for massive QED
- difficult part $\mathrm{d}\sigma^{(aux)}$ not needed

what we are doing now

- iron out last kinks in the muon decay
- implement electronic correction to $\mu - e$ at NNLO

what we will do soon

- study scheme dependence in FKS^2
- full NNLO for $\mu - e$