Zurich PhD Seminar 2018

Fully differential predictions for lepton decays

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Based on 1611.03617 and 1705.03782

Outline

motivation

radiative decay @ NLO

rare decay @ NLO

consideration @ NNLO

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motivation: lepton decays as...

... a SM process

- G_F is measured through the muon decay
- large-ish radiative corrections for measurements of $\tau \to e\nu\bar{\nu}\gamma$ and $\mu \to e\nu\bar{\nu}\gamma$
- radiative ($\mu \to e\nu\bar{\nu}\gamma$) and rare ($\mu \to e\nu\bar{\nu}e^+e^-$) decays are important backgrounds to searches for LFV

... a clean QED toy process to study

- efficient ways of IR subtraction
- regularisation scheme dependencies & γ^5 schemes
- derive two-mass fragmentation function to apply in other processes

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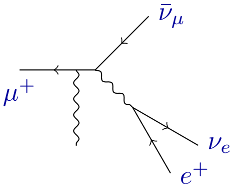
the radiative decay

- 4-Fermi interaction, fierzed at the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\textsf{QED}} + \frac{G_F}{\sqrt{2}} j_{V-A}(\mu, e) \cdot j_{V-A}(\nu_\mu, \nu_e)$$



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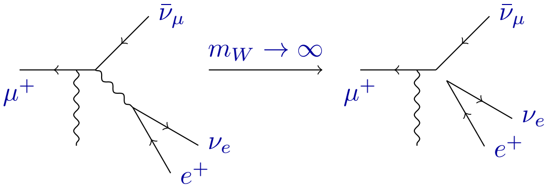

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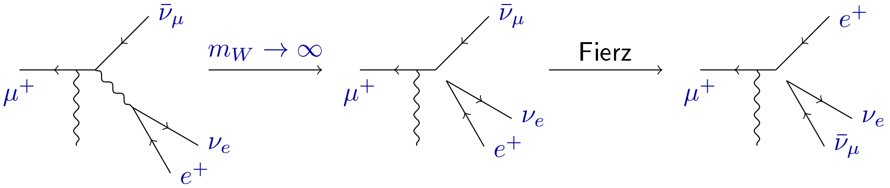

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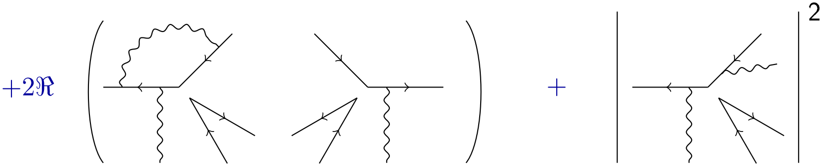
- can create arbitrary distribution with arbitrary cuts @ NLO



unfolding of acceptance

correcting for detector's kinematic acceptance is not trivial!

- $\tau \to e\nu\bar{\nu}\gamma$ @ BaBar: $3.5\,\sigma$ discrepancy between measurement and branching ratio NLO calculation [Fael, Mercolli, Passera 2015]

  $\Rightarrow$ potentially due to restrictive cuts

- $\mu \to e\nu\bar{\nu}\gamma$ @ PiBeta: $3.75\,\sigma$ discrepancy

  $\Rightarrow$ potentially due to mass effects $m_e > 0$

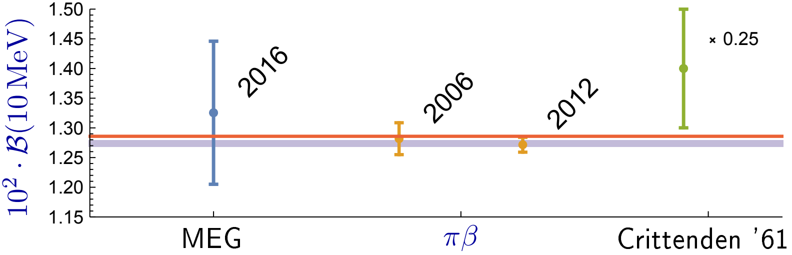
- we do not claim that any of this is the full solution!
- but $\mathcal{O}(10\%)$ QED effects are possible

global comparison: $\mathcal{B}(10\,\mathrm{MeV})$

- relate all data using NLO Monte Carlo to $E_\gamma > 10\,\mathrm{MeV}$
- compute kinematic acceptance ϵ

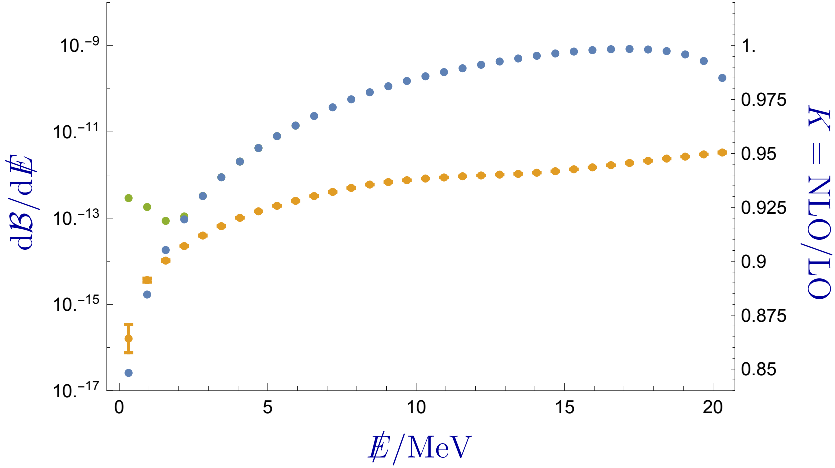
$$\mathcal{B}(10\,\mathrm{MeV}) = \underbrace{\frac{\mathcal{B}_{\mathsf{NLO}}(10\,\mathrm{MeV})}{\mathcal{B}_{\mathsf{NLO}}(\mathsf{exp.\ cuts})}}_{\epsilon} \mathcal{B}_{\mathsf{exp}}(\mathsf{exp.\ cuts})$$

- $\epsilon_{\mathsf{MEG}} \approx 2 \cdot 10^5$, $\epsilon_{\pi\beta} \approx 3$
- combined experimental $\bar{\mathcal{B}}(10\,\mathrm{MeV}) = 1.27(1) \cdot 10^{-2}$ (1σ above theory)



invisible energy spectrum at MEG

- $\mathcal{B}_{\text{NP}} \simeq 4.2 \cdot 10^{-13}$



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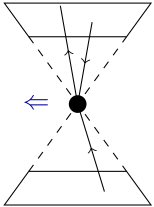
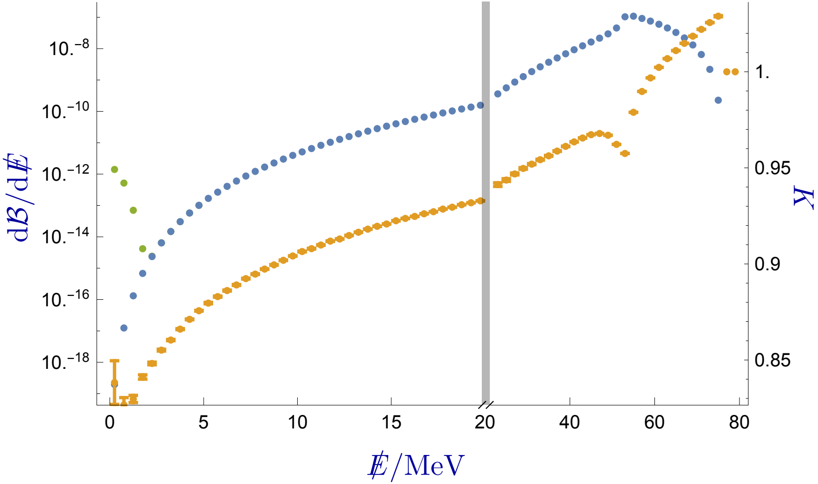
rare decay

- $4_{\text{Born}} + 40_{\text{1-loop}} + 20_{\text{real}}$ diagrams up to pentagons
- good parametrisation of phase space very important
- approximate Mu3e cuts $E_{e^\pm} > 10\,\text{MeV}$, $|\cos\sphericalangle(\vec{p}_{e^\pm}, \vec{e}_z)| < 0.8$
- calculated also for the four τ decays



invisible energy spectrum at Mu3e

- $\mathcal{B}_{\text{NP}} \simeq 10^{-12}$



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muon decay @ NNLO

- first NNLO calculation published 1999 (using optical theorem) [van Ritbergen, Stuart 1999]
- calculation of the energy spectrum in 2005 [Anastasiou, Melnikov, Petriello 2005]
- form factor (fully differential) only for $m_e = 0$ [Bonciani, Ferroglia 2008]

⇒ calculate only NNLO terms $\alpha^2(\ln m_e)^n$ by expanding the master integrals (strategy of regions)

- 'done' for the energy spectrum [Arbuzov, Melnikov 2002]

NNLO $\mu\, e$ scattering

- recent proposal to measure a_μ^{HLO} with $\mu\, e$ scattering
- requires theoretical uncertainties below 10^{-5}
  $\Rightarrow$ need NNLO-QED with $m_e > 0$
- problem: integrals with $m_e > 0$ are currently impossible for all intents and purposes

$$\mathcal{A}(t, m_e, m_\mu) \simeq Z_J(m_e) S(t, m_e, m_\mu) \tilde{\mathcal{A}}(t, m_\mu)$$

- jet function Z_J from muon decay, soft function $S(t, m_e, m_\mu)$ is 'trivial'

strategy of regions

- integrals for $\mu(p) \to e(q) + \nu\bar\nu$ with full mass dependence published recently [Chen 2018]
- does not allow extraction of Z and S $\Rightarrow$ use strategy of region instead
- write $p = p_+ + p_-$ and $q = q_- + q_\perp$ and identify regions h ($k \sim (1,1,1)$), c ($k \sim (\lambda^2, 1, \lambda)$) and s ($k \sim (\lambda, \lambda, \lambda)$)

$$\mathcal{I} = \int_{k_1\, k_2} \frac{1}{k_1^2 - 2k_1\cdot p}\,\frac{1}{k_2^2 - 2k_2\cdot q}\,\frac{1}{(k_1-k_2)^2}\,,$$

$$\mathcal{I}^{h_1\text{-}h_2} = \int_{k_1\, k_2} \frac{1}{k_1^2 - 2k_1\cdot p}\,\frac{1}{k_2^2 - 2k_2\cdot q_-}\,\frac{1}{(k_1-k_2)^2} + \mathcal{O}(m_e^2)\,,$$

$$\mathcal{I}^{h_1\text{-}c_2} = \int_{k_1\, k_2} \frac{1}{k_1^2 - 2k_1\cdot p}\,\frac{1}{k_2^2 - 2k_2\cdot q}\,\frac{1}{k_1^2 - 2k_1\cdot k_2^-} + \mathcal{O}(m_e^2)\,,$$

NLO warm up

- For $\mu \to e\nu\bar{\nu}$ we have $F(s,m_\mu,m_e) \simeq \sqrt{Z_J(m_e)}\,\tilde{F}(s,m_\mu)$:

$$F^{(1)}(s,m_\mu,m_e) \simeq \tilde{F}^{(1)}(s,m_\mu) \\ \qquad - \underbrace{\frac{\alpha}{4\pi} m_e^{-2\epsilon}\left(\frac{1}{\epsilon^2}+\frac{1}{2\epsilon}+\zeta(2)+2\right)}_{1/2\,\delta Z_J^{(1)}(m^2)} \tilde{F}^{(0)}(s,m_\mu)$$

  as expected from single mass case [Becher, Melnikov 2007]

- $S^{(1)}(s,m_e) = 1$ because there are no internal fermion loops
- only hard and collinear contribute at NLO
- For $\mu\, e$ scattering we have $\mathcal{M}(s,t,m_\mu,m_e) \simeq Z_J \tilde{\mathcal{M}}(s,t,m_\mu)$:

$$\mathcal{M}^{(1)}(m_\mu,m_e) \simeq \tilde{\mathcal{M}}^{(1)}(m_\mu) + \delta Z_J^{(1)}(m_e)\tilde{\mathcal{M}}^{(0)}(m_\mu)$$

  checked explicitly

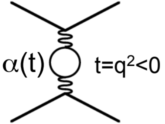
- method of regions and factorization works at NLO

conclusion / outlook

- fully differential NLO prediction are available for both $\ell \to l\nu\bar{\nu} + \gamma$ and $\ell \to l\nu\bar{\nu} + l^+l^-$
- radiative corrections can be extremely important when unfolding fiducial acceptance to 'PDG values'
- MEG & Mu3e: Corrections are negative, normally small (percent level) but can reach $\mathcal{O}(10\%)$
- all two-loop topologies for $\mu \to e\nu\bar{\nu}$ calculated

Alternative approach: a_μ^{HLO} from space-like region

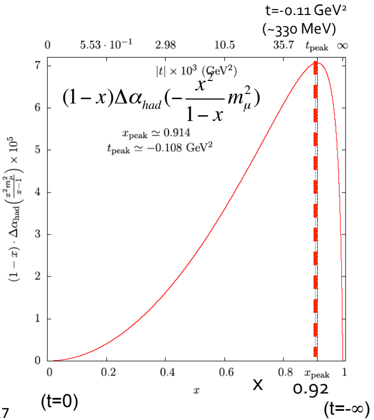
$$a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Delta\alpha_{had}\left(-\frac{x^2}{1-x}m_\mu^2\right)dx$$



$$t = \frac{x^2 m_\mu^2}{x-1} \qquad 0 \le -t < +\infty$$

$$x = \frac{t}{2m_\mu^2}\left(1-\sqrt{1-\frac{4m_\mu^2}{t}}\right); \qquad 0 \le x < 1;$$

- a_μ^{HLO} **is given by the integral of the curve** (smooth behaviour)
- **It requires a measurement of** the hadronic contribution to the effective electromagnetic coupling in the space-like region $\Delta\alpha_{had}(t)$ **($t=q^2<0$)**
- **It enhances** the contribution from **low** q^2 region (below 0.11 GeV²)
- **Its precision** is determined by the **uncertainty** on $\Delta\alpha_{had}(t)$ in this region



G. Venanzoni, μ-e Theory Workshop, Padova, 4 September 2017